As it is clear from figure 17, height of water and cola drops increased and height of syrup and salty water decreased during solidification. Because of vertical expansion of water and cola drops, we can observe point tip phenomenon with these two drops.

**Experiments results**

According to the fact that water and cola drops (with 0.28 and 0.39 Bond numbers respectively) had expanded vertically, and salty water and syrup drops (with 0.62 and 1.5 Bond numbers respectively) had radial expansion, and with attention to “Freezing Singularities in water drops” article, we can conclude that:

“If Bond number of a fluid is between 0.25 and 0.4 and the fluid expands during solidification process, we can observe a singular shape during it is freezing.”

**Conclusion**

In this article, according to the question, we studied how water and some other liquids freezes into a singular shape (pointy tip). As described in the article, to happen this phenomenon, there are two necessaries:

1-During solidification, drop should expand. (Because the volume of the drop should increase, so the drop will occupy more space and increased volume will appear as a singular point.)

2-This expansion, should be vertical. For vertical expansion happening, surface tension of drop should overcome its weight. This condition can be explained using Bond number (weight to surface tension ration) that we mentioned in this article.

So, we can say the following expression about water drop:

“When a water drop disposes on a cold surface, it starts to freeze from bottom (cold surface-drop interface). According to special property of water – It expands during solidification – volume of drop increases. There are two ways to expand: radial expansion and vertical expansion. Because the Bond number of water drop is between 0.25 and 0.4 so it is capable of vertical expansion. Thus the button section of drop expands vertically and remaining parts of drop (which are still liquid and are placed on the solid section of drop), without any transformation, will move upward. This happening will continue until we reach to the top of water drop. At the top point, the entire lower surface has been frozen. So the last point (top point) will move upward without any transformation, and the sharp point will be produced.”

**References:**

[1]- Pointy ice-drops: How water freezes into a singular shape by Jacco H. Snoeijer and Philippe Brunet.


---

**2014 Problem 12: Cold Balloon**

As inflated, a rubber balloon’s surface becomes cooler to the touch

**Abstract**

The surface temperature of a balloon would decrease during the deflating process, which originates from the characters of rubber. We construct a semi-quantitative model to investigate this phenomenon, and obtain the temperature of the balloon as a function of time theoretically with some parameters which need to be measured experimentally. And we verify the model though experiment. We also analyze factors that affect the cooling process, and find that thickness is particularly important. By measuring the thickness distribution of the balloon, we can predict the temperature distribution theoretically, as observed in the experiment.

**Fig.1:** (a) Measuring the temperature of a piece of rubber, we can clearly observe that the temperature will change when work is done by or on the rubber. (b) Fasten a stretched rubber band on two sides, and put an indicator to show the motion of middle point. (c) Heat its left side by an electric air blower. (d) Heat its right side.

**Wang Han Wei**

Tianjin Experimental High School, China

henry.w_Hwi@yahoo.com

**Zhang Ruo Yang**

Department of physics, Nankai University, China

zhangruoyang@gmail.com

**Keywords**

Balloon, deflation, Cooling, Thermodynamics of Rubber
Introduction

Balloon skin is made of rubber. The entropy of rubber molecules would increase when it contract [1], and the temperature of it would decrease. As the balloon skin does work to push the gas out of the spout, the temperature of the skin will decrease when the balloon shrinks, as shown in Fig. 1(a). Moreover, the force of rubber changes with its temperature. The hotter part would generate bigger force at the same stitch rate. Therefore, the rubber band’s middle point will move to the side with higher temperature, as shown in Fig. 1(b)-(d).

Unlike normal materials, rubber does not obey Hooke’s law. In real situations, the force given by rubber is not a certain function of its strain, since stretching the rubber can change the molecular arrangement (Mullins effect). But we can reduce the influence by using new balloons and finish all the experiments once. We assume that the tension per unit sectional area of a balloon is a certain function of the stretch ratio. Though the theory from the entropy formula cannot explain the facts, we fit the experimental results with a sixth polynomial, which leads to an approximate expression of the theoretical formula.

Theory of Cooling

As the balloon shrinks, rubber does work to the gas. To calculate the temperature of a certain position, we choose to analyze a piece of rubber with fixed thickness.

As we mentioned before, the temperature of rubber will change when it does work, because a certain percentage of work is translated from its internal energy. Let be this percentage which can be measured experimentally.

\[ T_R(T_E - T_R) = \frac{1}{2} \int P(t) \, dt \]

Fig. 2: Tension per unit sectional area versus stretch ratio (l/l0).

The curve fits the data with the expression.

\[ F = \frac{1}{2} \pi r P \]

Then the tension can be written as a function of radius and pressure.

To get the temperature of rubber balloon changing with time, all we need is the expression of stretch ratio \( \alpha(t) \) as a function of time. Assuming the balloon is a sphere, the equilibrium of a hemisphere gives

\[ \pi r^2 P = 2\pi r F \]

(4)

Then the tension can be written as a function of radius and pressure.

\[ F = \frac{1}{2} r P \]

(5)

Here, \( F, r, P \) are all varying with time.

According to the Bernoulli equation, the velocity of the air flow ejection out of the spout can be expressed with the relative pressure \( P_r \), however, a correction coefficient \( \lambda \) should be added to correct the loss of energy caused by gas turbulence and viscosity:

\[ v = \lambda \left( \frac{2\pi TR E}{M P_r} \right)^{1/2} \]

(6)

And \( \lambda \) can be measured by experiment. The relative pressure \( P_r \) and the radius \( r \) of the balloon have the approximate relation [3, 4]:

\[ P = 2 \pi r \left( \frac{1}{r_0} \right) \left( \frac{1}{r} \right) \left( \frac{1}{r} \right) \left( \frac{1}{r} \right) \]

(7)

And in terms of the rate of change of the volume

\[ \frac{dV}{dt} = \frac{\lambda}{4} + r \]

(8)

with \( \lambda = 4 \pi r^2 / 3 \), the pressure \( P(t) \), and radius \( r(t) \) as functions of time can be obtained according to Eqs. (5) - (7). \( \Delta r \) the initial thickness of the balloon skin, \( r_0 \) the initial radius of the balloon, \( r \) the radius of the balloon, \( s, ds \): coefficients which are influenced by the type of rubber material

Substituting \( P(t), r(t) \), and Eq. (2) into Eq. (3), we can figure out how the temperature of the balloon \( T_R \) changes with time for a given initial condition by measuring the corresponding parameters.

Experimental measurement

The experimental results shown in Fig. 4 indicate that a smaller balloon can produce a higher pressure. By fitting the experimental data with Eq. (7), we can see that the theoretical formula is consistent with the experimental results on the whole.

1. Pressure changing with radius

2. Proportion of absorbed heat to work

Let a piece of rubber stretch a slider and measure the final velocity of the slider and the temperature difference of the rubber. The kinetic energy equals to the work that the rubber does. And the difference of rubber’s internal energy \( \Delta E \) equals to the absorbed heat. Therefore, we can calculate the proportion

\[ \gamma = \Delta E/W. \]
experiments shown in Fig. 5, we obtain $\Delta \frac{E}{W} = 82.99 \pm 6.36\%$ which shows that the error of this method is too big to obtain an accurate result.

Hence, we choose another method to measure this percentage [2].

The reason of absorbing heat is the change of entropy while rubber does work. The work $dW = Fdl$ and the entropy difference have the relation:

$$dE_p = dW + TdS$$  \hspace{1cm} (9)

Therefore, the tension can be written by $F = \frac{dE_p}{dl}, S(\dot{l})$. Supposing that the entropy is a function of the deformation, $S(\dot{l})$, then the internal energy is also a function of one variable $\dot{l}$, and the tension can be expressed as a linear function of temperature:

$$F = \frac{dE_p}{d\dot{l}} \frac{d\dot{l}}{dT} T$$  \hspace{1cm} (10)

The experimental data shown in Fig. 6 agree with this relation. By fitting to the experimental points, we can get the two parameters $B = \frac{dE_p}{dl}, A = \frac{dE_p}{dT}$. Accordingly, the proportion can be computed by

$$\gamma = \frac{\Delta T}{W} = \frac{-A T d\dot{l}}{(AT + B) d\dot{l}} = \frac{-A T}{AT + B}$$  \hspace{1cm} (11)

Eq. (11) shows that the temperature will influence this proportion (see Fig. 7), however, since the temperature only changes within 5 degrees and is much smaller than the environment temperature which is about 300 K, the value of $\gamma$ can be taken at 300K with omitting the temperature shift.

$$\gamma \approx 88.48 \pm 0.13\%$$  \hspace{1cm} (12)

3. The correction coefficient of Bernoulli equation

Fig. 8 shows the relative pressure inside the balloon varying with time. The pressure is stable during 3-7 seconds, the rate of change of the pressure is smaller than 12.3 Pa/S. Therefore, the velocity of jet out of the balloon is approximately stationary. In terms of the rate of change of the volume and Eq.(6), the correction coefficient can be determined. Then we obtain the phenomenological expressions of the relative pressure and the radius of the balloon changing with time.

4. Coefficient of heat transfer

The coefficient of heat transfer is obtained through measuring the warming process after the cooling (see Fig. 9).

For the cooling process, the heat transfer is very fast at the beginning, because it happens to both inside and outside of the balloon, but as the interior gas cooling down, the temperature inside the balloon gets close to the balloon surface, and the heat transfer to the inside levels off to zero. We assume the heat transfer only happens to one side.

Now, we’ve got all the parameters we need to compute the temperature of the balloon changing with time. Solving Eq(3). We obtain the theoretical result of the temperature changing with time. As shown in Fig. 10, the theoretical result agrees well with the weighted average temperature of experiment.

Influence of the parameters

Considering the heat transfer, the longer the process is, the bigger the influence would be. So a smaller spout will produce a slower volume flow rate and will lead to a smaller final temperature difference as shown in Fig. 11(a).

On the other hand, as absorbed heat by doing work is the main cause of cooling, now we focus on this factor but ignore the heat transfer which can only weaken the cooling. In this case, the temperature difference between the balloon and environment is only influenced by the stretch ratio:

$$\Delta T = \frac{2 \epsilon}{C_p \rho v} \int_0^{\infty} (\alpha) d\alpha$$  \hspace{1cm} (13)

On account of $\epsilon (t \rightarrow \infty ) \rightarrow 0$, Eq. (13) indicates
that the initial stretch rate is the only factor that influence the final cooling temperature. In addition, the stretch rate is determined by the initial size and initial thickness of the balloon. The larger initial size, the higher initial stretch ratio, therefore, the lower the final temperature difference would be. This qualitative conclusion will still hold, more accurately, if the heat transfer is also taken into account, which can be verified by both experiment and theoretical calculation with Eq.(3), as shown in Fig. 11(b).

**Temperature Distribution**

Although rubber doesn’t obey the Hooke’s Law, the tension per unit sectional area $f(\alpha)$ is also positive correlated to the stretch rate $\alpha$. Thus for a fixed tension $T = f(\alpha)\rho d$, the thicker the balloon is, the smaller the initial stretch ratio $\alpha$ would be. Assuming the balloon maintains spherical, the tension would be the same for different positions.

Therefore, the thinner part with a bigger initial stretch rate would be colder in the end according to Eq. (13). In Fig. 13, we can clearly see that the top of balloon (thinner part) is colder than the bottom (thicker part).

Moreover, as the slope of tension with respect to stretch rate is not a constant (see Fig. 14(a)), assuming the tension decreases with a constant rate (This approximates to the real situation, because the pressure is stable for most of the time of the whole process), and temperature would decrease faster for the period with a lower slope. But the parts with different thickness experience this quick cooling period at different time. The thicker part experiences this earlier. To simplify the problem, we ignore the heat transfer. And solving Eq. (13) with a fixed initial tension, we get the temperature as functions of time for different positions as shown in Fig. 14(b). Experimentally, we observe that the thicker part first experiences the fast cooling period, but the final difference of temperature is lower than the thinner part, as shown in Fig. 14(c)-(e). The infrared photographs shown in Fig. 14(d)-(e) exhibit clearly that the coldest area moves upward. These observations are consistent with our qualitative analysis. However, comparing Fig. 14(d) and Fig. 14(e), this phenomenon is more apparent for a bigger balloon, because the influence of heat transfer from the air is smaller.

We can track a certain position by marking it with water. Water will evaporate and form a low temperature mark which can be observed from the infrared photograph. As shown in Fig. 14(f). The temperature differences for point 5 and 6 decrease before the end of the leaking process which is different from the previous theory. The reason of this is heat transfer from air.

**Conclusion**

In this paper, we investigate the interesting phenomenon that the temperature of a balloon will decrease while deflating both by theory and experiment. We also have explained the temperature distribution on the balloon surface observed in experiment.

We found the main reason of cooling is rubber’s special characteristic that it absorbs heat while doing work, and the temperature difference is in positive proportion to the change of stretch rate. The thickness of the balloon skin is the main parameter which influences the cooling, as the stretch rate’s function of tension force is mainly determined by it. Moreover, the nonlinear force function of stretch rate make the temperature distribution various with time.

**Acknowledgement**

We special thank the following persons who gave us assistances and supports in different aspects: Geng Heming & Li Guotian, Pro. Ye qing & Pro. Feng ming, Pro. Wang Yang, Ju Zhiyong.

We also thank the Tianjin University, Nankai University, and Tianjin experimental high school for lending the experiment equipment and labs.

**Reference**


It has been seen that when we release the gap of the air filled balloon, the air moves out and the balloon get cooler to the touch.

The problem is why the balloon gets cooler and what is the temperature of the each point of the balloon as a function of effective parameter?

Theoretically we will prove that the balloon and also the air inside will get cooler and we have gradient temperature on the surface of the balloon.

As you can see in this article finding the heat that moves from balloon to the air is easy, the real challenge is finding the temperature of each point of the balloon. Due to this at first we make some assumptions and solve the problem using the assumption and then for making our model closer to the reality and what we seen in our experiments, we had tried to decrease our assumptions and we had been forced to use numerical solution to find the answer.

Introduction – Temperature decreasing

Due to the first law of the thermodynamic we have the relation between internal energy, heat and work for a thermodynamic system. When we push the air inside the balloon the air receive negative work then the internal energy will increase after that due to thermal equilibrium the temperature of the air will decrease until it reaches to the ambient temperature. Now we release the gap, the air moves out and it had done work. So the internal energy decreases again. And we reaches to a situation that the temperature is less than the ambient temperature.

This will occur during the time that the air moves out. So in this time the balloon’s surface temperature is more than the air’s, and