2011 Problem 8: Domino Amplifier

A row of dominoes falling in sequence

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Abstract

We talk about the general shape of the dominoes, according to the characteristics of its geometry and explore the energy and momentum transformation process of two or a row of dominoes. By comparing the experiment results, simulation is consistent with actual process. Theoretically, the energy amplification is almost with no limitations.

Keywords

dominoes, energy and momentum transfer process, energy amplification

Introduction:

Let's review the topic: A row of dominoes falling in sequence after the first is displaced is a well known phenomenon. If a row of "dominoes" gradually increases in height, investigate how the energy transfer takes place and determine any limitations to the size of the dominoes.

Dominoes' limited sizes are length, width, height, distance and they relate to domino magnification. A row of dominoes' size only increases in height or zoom proportional in turn. We also need to explore the relations of energy transfer and amplification.

First of all, we all know that domino fall is a pure rotation process. Let's talk about the inevitability of this exercise form. There is a certain static friction f_0 or dynamic friction f between the dominoes and ground. If we exert horizontal force F on the domino bottom and $F > f_0$, domino will move in horizontal plane. This case doesn't belong to our research theme scope. If $F < f_0$, dominoes rotate around the contracting point only. Under external force, the first domino's center of gravity is pushed to the highest point. When it falls down, its gravitational potential energy turns into rotational energy. And when the first collides with the second, its angular momentum transfers to the next through collision.

Except the first domino, every domino's rotational energy is in addition to the forward domino release of gravitational potential energy, but also "inherit" all the prior dominoes' released kinetic energy. We can summarize the energy transfer process using an equation

$$E_{total}=T+E_c+E_f$$

where E_{total} is all falling down dominoes' released gravitational potential energy. T is the next domino's inherited kinetic energy, the release of energy, E_c is collision loss energy, when one is sliding down and E_f is surface friction loss energy. We define

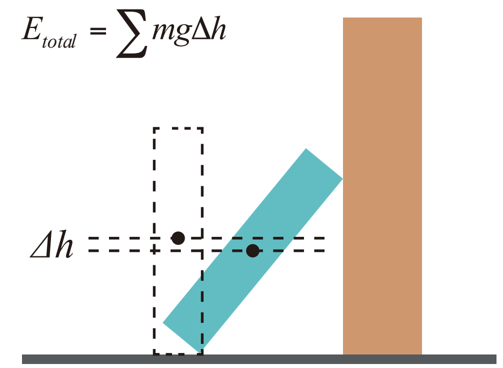


Fig 1. The block released gravitational potential energy through its center of gravity dropped Δh.

Where m is the mass of domino, g is acceleration of gravity, Δh is the domino's change in height of the center of gravity in Fig 1.

Vertically standing domino has an initial gravitational potential energy. Part of the energy transfer occurs in two neighboring dominoes' collision. The collision stimulates the release of gravitational potential energy. Pure energy amplification increases the release of gravitational potential energy in turn.[1]

We made model approximation: block's bottom friction is large enough, so dominoes rotate around the contracting point only; Different blocks and different locations of one domino's density are equal. Because each collision is an incompletely elastic one, compared with the translational collision, we introduce a parameter e_a, the coefficient of restitution. We'll define and illustrate e_a later.

Simulation:

A.A row of dominoes are zooming in turn proportionally

We define dominoes' thickness, height, width as a, b, c, distance between first two dominoes as d, geometric amplification coefficient as k. Adjacent blocks' geometric relationship is shown in Fig 2.

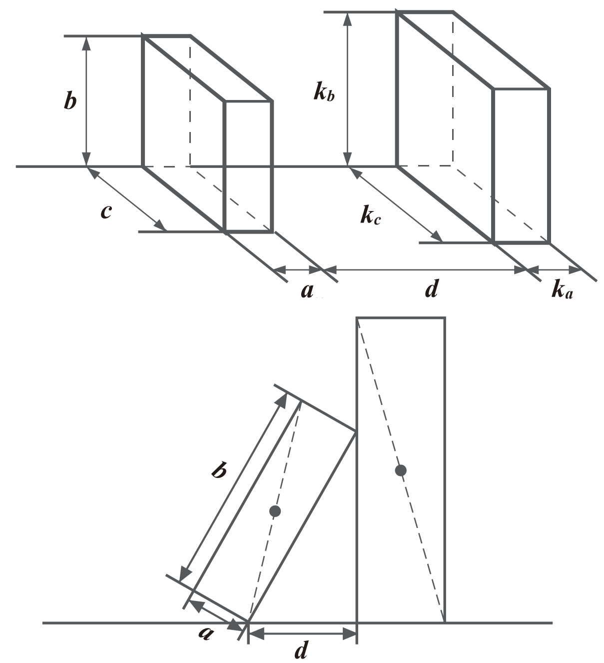


Fig 2. a, b, c and d geometric relationship

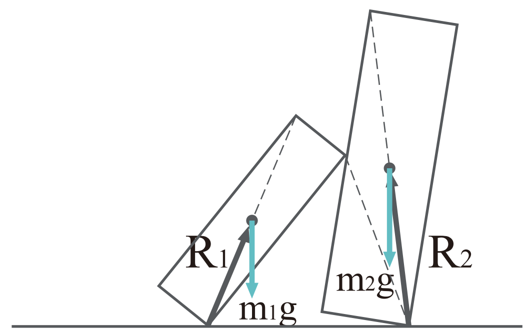


Fig 3. Two blocks gravity torque balances relative positional relationship.

Adjacent two dominoes have a certain relative position when their gravity torques are balanced.

$$m_1\vec{g}\times\vec{R_1}+m_2\vec{g}\times\vec{R_2}=0$$

Their positional relationship is shown in Fig 3. At the equilibrium position, if their total non-zero angular momentum vector points within the paper, the next domino will be pushed down.

Next, we do some theoretical analysis. Define the ith block's mass, density as m_i, ρ_i. The gravitational potential energy turns into rotational energy when the first domino fell down from the highest point. The second block obtains part of front one's rotational energy to provide the increase in its gravitational potential energy through collision process. We introduce the center of gravity collision coefficient of restitution e_a. Its physical meaning is: measure the degree of rigid objects collision coefficient. Taking into account the shape factor of a domino, we introduce this equation:

$$e_a=\frac{1+e}{R}$$

where R is a shape parameter:

$$R=1+\frac{\sqrt{b^2-d^2}+\mu d}{\sqrt{b^2-d^2}-\mu ka} \qquad [2]$$

If e_a=1, the collision process is completely elastic collision, and if e_a=0, it is a completely inelastic collision. μ is friction coefficient between domino's bottom and the horizontal plane. Collision process can be written as equation

$$I_{2\omega 2}=e_a\cdot I_{1\omega 1}$$

where I is the Rotational Inertia, ω is angular velocity. Knowing μ, we can quantitatively calculate the energy loss during collision process. These are shown in Fig 4. Now, we get some main equations.[2][3]

$$\frac{1}{2}m_1 g\left(\sqrt{a^2+b^2}-\sqrt{b^2-d^2}-\frac{ad}{b}\right)=\frac{1}{2}I_1\omega_1^2$$

$$I_2\omega_2=e_a I_1\omega_1$$

$$e_a=\frac{1+e}{R}$$

$$R=1+\frac{\sqrt{b^2-d^2}+\mu d}{\sqrt{b^2-d^2}-\mu ka}$$

$$\frac{1}{2}km_2 g\left(\sqrt{a^2+b^2}-b\right)=\frac{1}{2}I_2\omega_2^2$$

There are 11 roots if we solve equations. These roots are all possible values of k. The physical meaning of k is that when fixing parameters a, b, c, d (to a certain take shape domino), it can push down the largest one whose size is k_a, k_b, k_c. The root that we wish is a positive real number and there is only one root satisfies our request. According to the value of k we can know the limitation of magnification.

According to the above equations and a=2.00cm, b=10.00cm, c=5.57cm, we solve them, get the positive real number k, and plot the d-k curves. We get the relationship between k and two dominoes' distance d in Fig 4.

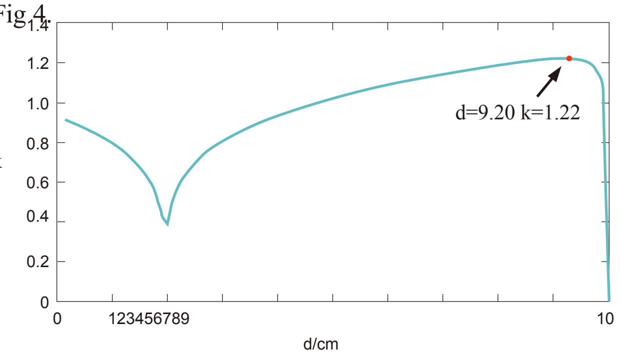
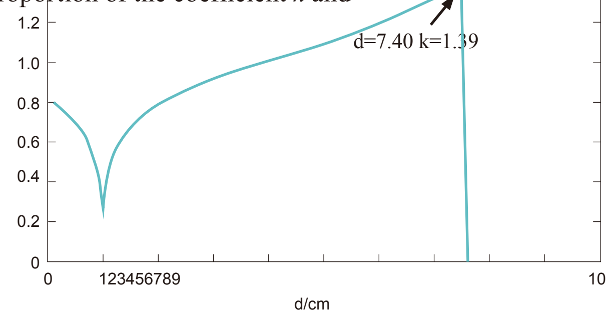


Fig 4. d-k curves in two successive dominoes model

Also we get the relationship between the largest proportion of the coefficient k and



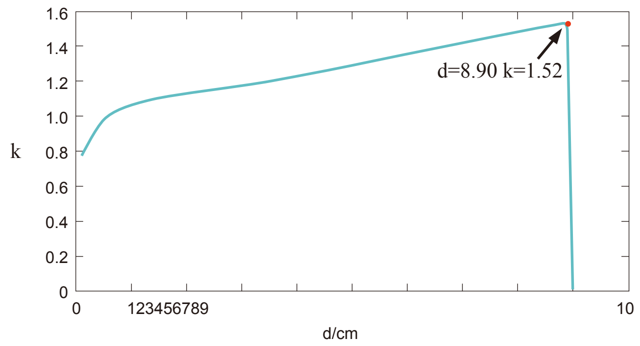


Fig 5. d-k curves in three successive dominoes model

three or four successive dominoes' distance d in Fig 5 and Fig 6.

There is one red dot in each picture. This point shows the maximum value of k in this case. In the above three models take the maximum value of k, the d values are 9.20cm (two successive dominoes), 7.40cm (three successive dominoes), 8.90cm (four successive dominoes). The results show that only at a specific distance domino could be successfully pushed down to achieve energy transfer. If the results are consistent with the experiment, k value can represent the amplifier limitation of dominoes.

B.Different dominoes only increase in height.

In part A, we discuss the case zooming proportionally in turn, there is another case that dominoes are increased in height only. Similar to the previous case, we can use the same equations, except that any two successive dominoes' moment of inertia is no longer a simple proportional relationship. If the forward domino's height b is 100.0mm, we can get the following results: the relationship between the largest proportion of the coefficient k and two successive dominoes' distance d, thickness a in Fig 7.

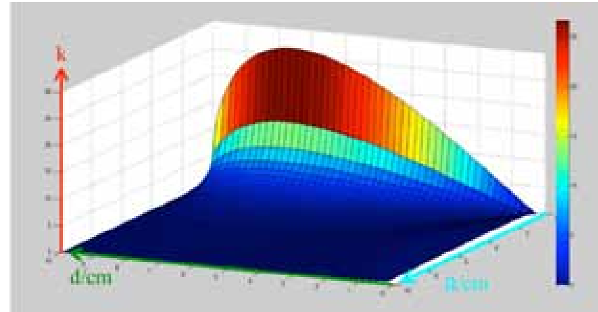


Fig 6. a, d andk curve surface which only increasing in height

We also apply the equations in part A. According to derived domino size in Form 2:

Domino number	a(cm)	c(cm)	b(cm)	$k_i=b_i/b_{i-1}$
1	1.20	2.50	3.75	
2	1.20	2.50	3.92	1.05
3	1.20	2.50	4.57	1.17
4	1.20	2.50	6.63	1.45
5	1.20	2.50	15.88	2.40

Form 1. The theoretical model parameters value

We can see k_2=3.92/3.75=1.05 that two adjacent dominoes height increases ratio belong to the 1st and the 2nd, but k_5=15.88/1.45=2.40 belong to the 4th and the 5th. If we calculate it between the 6th and the 7th, we will find k7=2612.69/98.24=26.59. With the certain thickness and only increasing height, k is increasing rapidly and the energy amplification is amazing. Theoretically, if domino and plane's condition is invariable, the ratio of height over thickness is with little limitation, the energy amplification is almost with no limitations.

Experiment

A.A row of dominoes are zooming in turn proportionally

In order to ensure that block bottom friction is large enough, we manufacture dominoes with wood and use sandpaper as the horizontal plane material. All the dominoes are grinded from the same piece of wood whose density ρ=429*kg/m3*. There isn't a series of standard domino size, so we selected a group of common domino dimensions whose thickness: width: height= 1:3:6 to grind. The tools and material used in experiment are shown in Fig 7

Fig 7. Tools and dominoes used in experiment

A series of dominoes are actually made one by one because each block is made under the condition that it could exactly push down the next higher one. So these experiment results apply to two successive dominoes, three successive dominoes, four successive dominoes simulation model.

We select the first block thickness, width, height as 0.28cm, 0.78cm, 1.40cm. Every next block's size is the largest one which can just be pushed down. There are some pictures which are printed screen in experiment video in Fig 8.

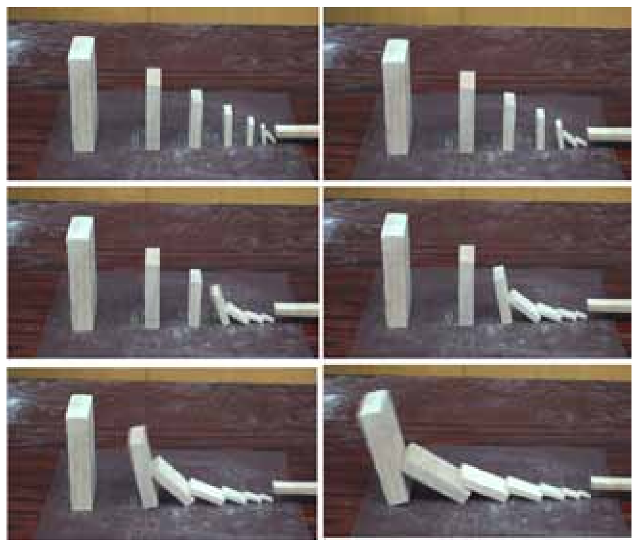


Fig 8. Printed screen of experiment video

Ki are defined as the equation below. There are the sizes for 7 blocks in Form 2.

$$\overline{k}_i = \frac{1}{3}\left(\frac{a_i}{a_{i-1}} + \frac{b_i}{b_{i-1}} + \frac{c_i}{c_{i-1}}\right)$$

Domino number	a(cm)	c(cm)	b(cm)	$\overline{k}_i$
1	0.28	0.78	1.40	
2	0.39	1.09	1.96	1.397
3	0.55	1.53	2.74	1.404
4	0.77	2.14	3.84	1.400
5	1.08	3.00	5.38	1.402
6	1.51	4.20	7.53	1.399
7	2.11	5.88	10.54	1.399

Form 2. Dominoes size in experiment

The results are: k=1.400±0.004. Theoretically, limitation for two successive dominoes k is 1.29, for three successive dominoes k is 1.41. The results that obtained are consistent in experiment with the model we build.

B.Different dominoes only increase in height.

In this case, it is hard to keep it well vertically standing with the large b over a ratio block. So we only use 5 blocks in this experiment. Each block is also made under the condition that it could exactly push down the next higher one. In experiment, k_i is larger than calculation result. On one hand blocks are too small, they might be imbalanced, on the other hand the simulation model is too simple. Printed screen pictures are shown in experiment video in Fig 9.

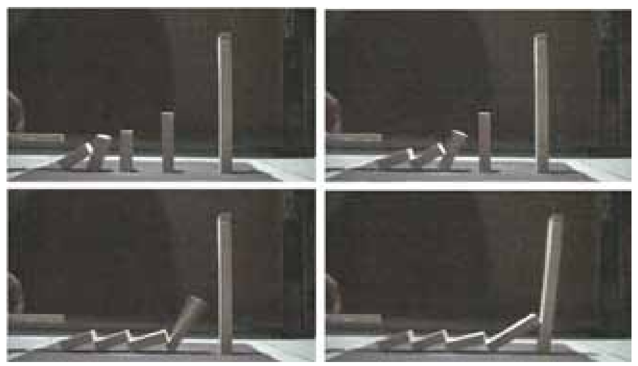


Fig 9. Printed screen of experiment video

There are several experimental errors. In the model we calculated the coefficient of restitution e as 0.5, but maybe there is a little deviation between wood coefficient and 0.5 in this experiment. If e is larger, the energy transfer efficiency is higher, so k is larger. Two dominoes sliding friction is ignored and the objects in experiment are too small, thus any hand shake will affect results. Different blocks, or different locations of one domino's density may not be uniform. These circumstances lead the model to being discrepancy in the experiment.

$$m_7/m_1 = \left(1.4^3\right)^6 = 4.3\times10^2$$

Conclusion

1. Size limitations:

A series of dominoes are of the same geometry, only to change the magnification factor k. For our study determined the thickness, width and height of the domino, the scale factor k=1.4, under this case one can just tear down the next. In particular for the magnification of the extreme point of k only at certain distance d will be pushed down.

2. Energy amplification:

Experiments include 7 dominoes, and the difference among them is mass and the energy released to represent. The rate of energy amplification is

$$E_{total_7}/E_{total_1} = \left(1.4^3\right)^6\times1.4^6 = 3.2\times10^3$$

Acknowledgements

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Reference

[1] W.J.Stronge and D.Shu. The domino effect: successive destabilization by cooperative neighbours. Proc. R. Soc. Lond. A.. 418, 155-163 (1988).

[2] W. J. Stronge .The Domino Effect: A Wave of Destabilizing Collisions in a Periodic Array . Proc. R. Soc. Lond. A 1987 409, 199-208(1987).

[3] Zhao Kaihua. Luo Weiyin. Mechanics[M]. Higher Education Press. 2nd edition (December 1, 2008).