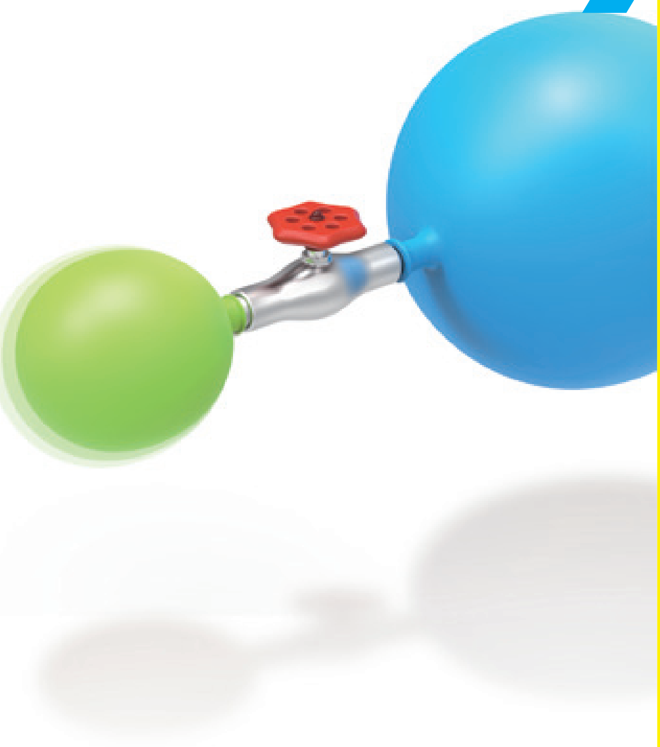

2015 Problem 5 : Two Balloons

Two Inflated Balloons Can Reach to a Stable Point by an Uncommon Air Flow

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Abstract

Two rubber balloons are partially inflated with air and connected together by a hose with a valve. It is found that depending on initial balloon conditions, the air can flow in different directions. We found that there are more parameters involved in this phenomenon such as Mullin effect and the pressure can influence on the direction direction without a linear relation with the volume. Also, depending on amount of air in each balloon, the potential energy is minimized so the final size of each balloon can be predicted. Usually, people assume that the balloon with higher volume will shrink into the smaller one but it doesn't always happen. When the valve is open, there are three possible results. First, the bigger balloon shrinks in smaller balloon, second, no movement happens and the most interesting result, the smaller balloon shrinks in bigger balloon. The movement is very fast and usually happens in few second (on average, five or six seconds). If the experiment repeats with used balloon, the result will change. The results of the experiments and the proposed theory are fitted and shows an expectable consistency. Two constants are measured, namely shear modulus and elastic coefficients, for incompressible non-linear elasticity. Shear modulus and the elastic coefficient are calculated of 0.25 N mm^{-2} and 3265 Pa, respectively. The elastic coefficient for calculating the pressure inside the balloon is 3265 Pa.

Introduction

Rubber balloons that usually adorn children's parties, can also enchant scientists. Indeed, there is much more than fun and games to be had with balloons; they have formed a suitable subject for mathematical studies and an interesting paradigm of methods of modeling in physics and chemistry. First of all, balloons which are consist of rubber, a remarkably unique elastic material with an extreme extensibility in comparison with solids. Over the past three decades a considerable amount of efforts have been directed towards finding the constitutive relation for rubber-like materials. Recently, a new constitutive relation based on experiments has been performed [2]. It has found that tensile instability (for a spherically symmetric balloon) is an important factor by applying the mentioned constitutive relation [3].

During inflation, due to increasing the radius of curvature and decreasing the thickness of the balloon, the inflating pressure needed to maintain at equilibrium reaches a local maximum. After dropping to lower values, the equilibrium pressure again starts to rise at very large deformations. This "Strain hardening" of the material is due to increase of crystallization, for minimizing the energy stored in the whole system, which appears to overcome the radius of curvature and thickness effects. This type of tensile instability is familiar to the child who has blown up a toy balloon. After developing the governing equations for the inflation of a spherically symmetric balloon produced from an elastic incompressible rubber-like material, the inflation of an initially spherical balloon is investigated both analytically and experimentally.

Pressure is defined as force per area, so in an equal area (like Two balloons experiments) air moves from high pressure space to low pressure space. Because of that, in some situation pressure versus volume is not linear, air can move from smaller space to bigger space. Therefore, finding the relation between pressure and volume is necessary to find the direction of movement. By P-V diagrams (pressure versus volume) we can find the direction of air movement. So there are some observations to find rubber's characteristics and behavior. We study equilibrium, ideal gas law, minimum of potential energy and pressure in rubbers.

Theories

Some type of permanent structure is necessary to form a coherent solid and prevent liquid like flow of elastomer molecules. This requirement is met by incorporating a small number of intermolecular chemical bonds (crosslinks) to make a loose three-dimensional molecular network. Such crosslinks are generally assumed to form in the most probable positions, so that the long sections of molecules between them have the same spectrum of end-to-end lengths as a similar set of uncross linked molecules would have. Under Brownian motion of each molecular section takes up a wide variety of conformations, as before, but now subject to the condition that its ends lay at the crosslink sites [4].

Two balloons reach their equilibrium, when the pressure inside them is equal. Hence, there would not be any further flow. Considering Newton's second law $P_A = P_B$ and Ideal gases when there is no flow between two balloons:

$$\frac{N}{n_B} - 1 = \left(\frac{\lambda_A}{\lambda_B}\right)^3 \quad (1)$$

where N is number of gas moles in both balloons, n_B is number of gas molecules per mole in balloon B and also λ is the radius per initial radius of balloon. From the mentioned equations we get:

$$P + P_0 = \frac{4h_0C_1}{r_0}\left(\lambda^{-1} - \lambda^{-7}\right) \quad (2)$$

where h_0 is unstrained thickness, r_0 is unstrained radius and C_1 is a coefficient which is proportional to the number N of molecular strands that make up the three-dimensional network and Shear Modulus. C_1 and C_2 (another constant) are elastic coefficients with a sum $2(C_1 + C_2)$ equal to the small-strain shear modulus G [4]. Therefore, pressure in balloons can be calculated. Evidently, every system wants to reach its minimum of potential energy; the potential energy in thin walled spherical rubbers is:

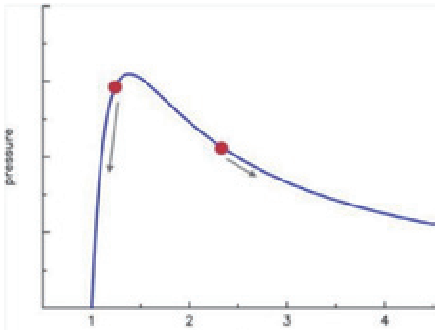


Diagram 1. for James gutch stress strain materials

$$W = G(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)/2 \qquad (3)$$

Where λ an extension ratio in three principle directions and G is accidentally shear modulus. So by these equations the pressure in equilibrium can be derived.

There are different theories for P-V relation by different assumptions for material stress-strain type and different molecular networks. If stress-strain assumption is the same as James-Gutch relation, the P- $\frac{r_0}{r}$ or P-λ is as diagram 1 & 2:

$$p_{(r)} = \frac{c}{{r_0}^2 r}\left(1 - \left(\frac{r_0}{r}\right)^6\right) \qquad (4)$$

Where, r_0 = Initial radius of both balloons, d_0 = Initial Thickness of Balloon, c is the experimental constant, s=Elastic Coefficient. But if the stress–strain relation of rubber is the Mooney–Rivlin type, the analytic form of the $P(r)$ relation is (diagram 3):

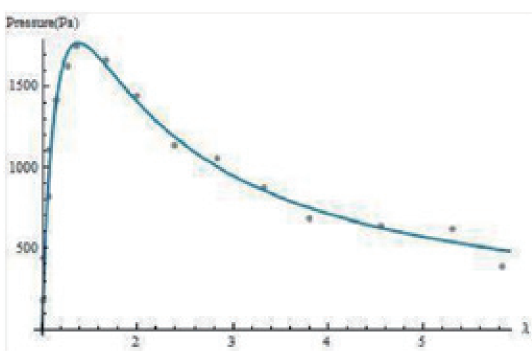


Diagram 2. Pressure vs. Extension. At first by inflating air pressure increases and reaches a peak but the volume increases hence pressure decreases.

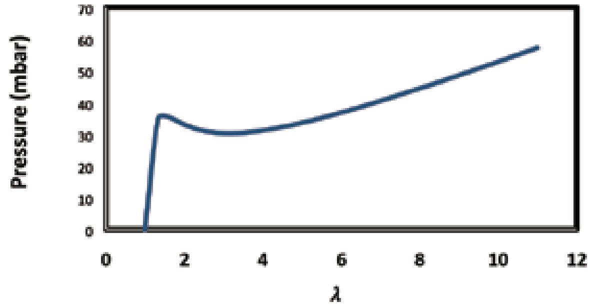


Diagram 3. Pressure versus

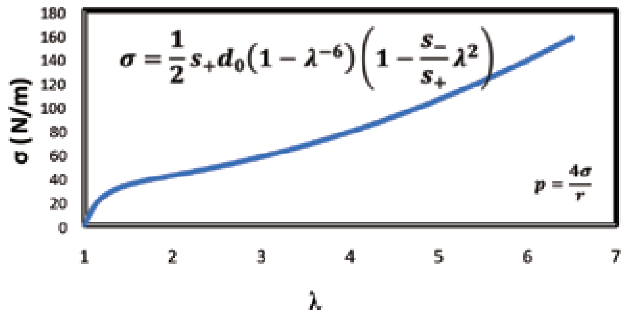


Diagram 4. Rubber Behavior

$$P = 2s_+\frac{d_0}{r_0}(\lambda^{-1} - \lambda^{-7})(1 - \frac{s_-}{s_+}\lambda^2) \qquad (5)$$

S_1 and S_{-1} are the two constants of a Mooney–Rivlin material. Diagram 4 shows the rubber behavior as:

$$\sigma = \frac{1}{2}s_+d_0(1 - \lambda^{-6})\left(1 - \frac{s_-}{s_+}\lambda^2\right) \quad p = \frac{4\sigma}{r} \qquad (6)$$

Experiments

In different balloons P-V relation is different so, experiments are necessary to find the exact relation and graph. In our experiments we tried to find the P-V curve exactly and derive some phases that show the direction of the movement. We use several balloons to find the exact

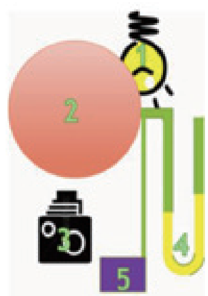


Fig.1. Experiments setup

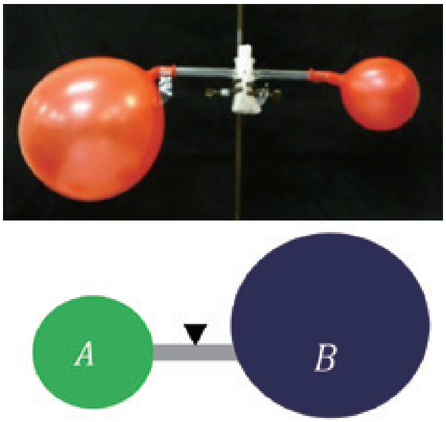


Fig.2. Setup for two connected b alloons

P-V relation then used image processing. The main idea for setup is a balloon connected to a manometer and a camera take video from inflating and flatting of balloon (Fig. 1). After that an image processing can show height of liquid in each frame. Beside volume of balloon was calculating in each frame so the pressure of each volume in every frame can be found. As in figure 4, a lamp used behind the balloon to minimize shadow to exact the image processing (1), a red balloon to simplify recognized border of balloon-area (2), a camera which

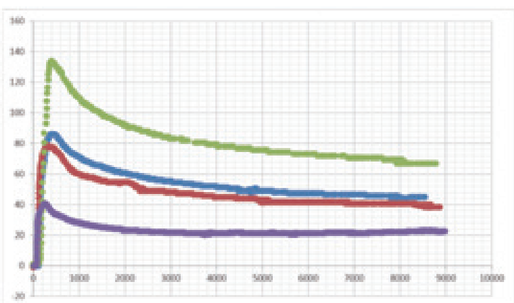


Diagram 5. Inflating series before interpolation

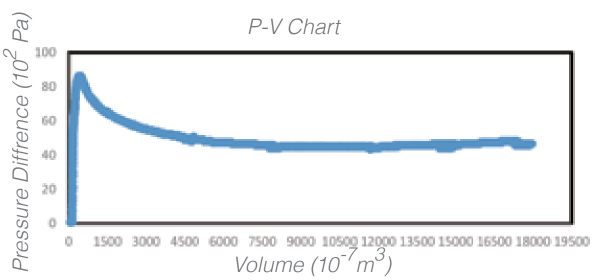


Diagram 6. Inflating series after interpolation

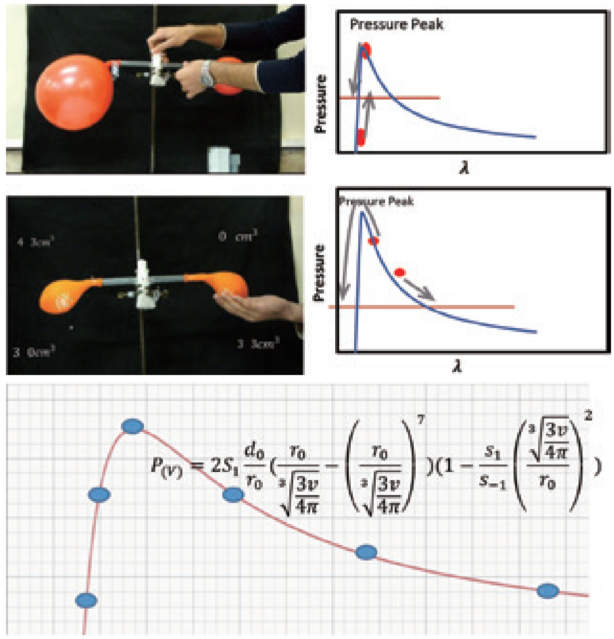


Diagram 7. P-λ: in two small and big balloons connected to each other. Red points are related to the initial condition of two balloons.

is installed perpendicular (3), a manometer with different color to find pressure (4), and a pump (5).

Setup for two connected balloons was used with a hose which has a valve (Fig.2)

Diagram 5 and 6 shows inflating series before and after interpolation respectively.

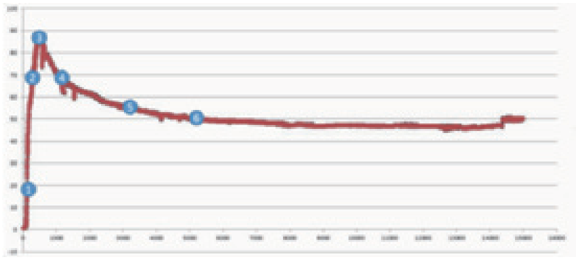
To compare different shapes after shrinking bigger balloon to smaller one and vice versa, P-λ diagrams are plotted in our experiments(Diagram7).

Results

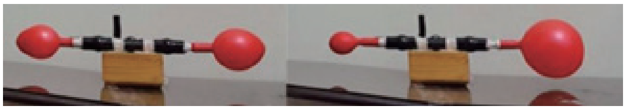
By fitting experiment results and Equ.7:

$$\frac{F}{A} = G\left(\lambda - \frac{1}{\lambda^2}\right) \qquad (7)$$

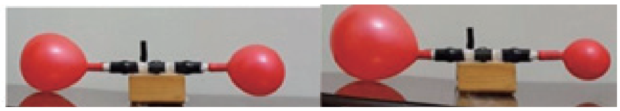
which F is force applied on Rubber and A is the area (thickness*length) [4], coefficient G have been calculated of 0.25 0.07 ± N/mm^2. Also by fitting experiment data and Eq. 2, $4c_1h_0/r_0$= 3265 Pa.



(a)



(b)



(c)

Fig.3. Points to show direction of movement in connected balloons

conculsion

The experiments and theories give us pressure-volume relation. According to them we can find phases show direction of movement (Fig. 3a). Six points which are chosen on the graph and phases are defined between them. Point 1 shows linear properties but points 2 and 4 are chosen because they have same pressure in different volume. Point 3 is the peak with maximum pressure and Points 5 and 6 show almost constant properties. Different parts of P-V graph have different Properties and this difference cause different velocity of discharging (diagram 8). In our experiment in points 1 and 2, we have big to small movement (fig 3b) and in experiment with point 5 and 3 we have small to big movement (fig 3c).

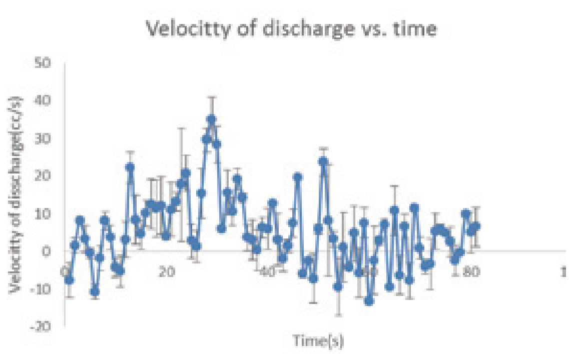


Diagram 8. Velocity of discharge versus time

The direction of movement can be found in each volume from our exact p-v relation. The velocity of movement as an important parameter was investigated in this problem.

So by taking a look at the theory we describe the airflow in two connected balloons. For the balloons which the pressure inside them has not reached its maximum yet, air flows from the larger one to the smaller one so changes of volume for them is not specific. Same happens for the balloons at the second part of the diagram (After the peak of diagram) but in this case the volume changes are great. However, when the smaller balloon is at the first part of diagram and the pressure inside of it is greater than the bigger one, which is at the second part of the diagram, so air, flows from the smaller to the bigger one. Furthermore, the most expected flow at the first sight (airflow from the bigger one to the smaller) occurs when pressure inside the larger balloon is greater than the smaller one. This phenomenon was investigated and the coefficients were measured. Airflow was described in different sizes and conditions for balloons. Trying balloons with different elasticity coefficients not always results the same as explained. This system (two connected balloons) like every other system reaches its minimum of potential energy, which is why balloons never burst in this phenomenon.

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