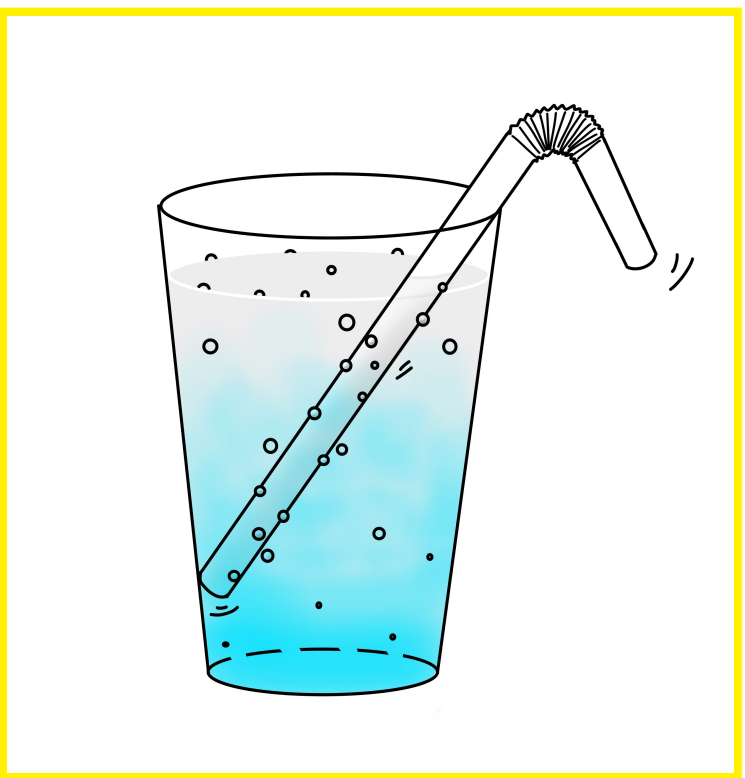
2018 Problem 5: Drinking Straw

Zheyuan Si, Yuan Wang, Chuanyong Li,Hua Yu*

School of Physics, Nankai University, Tianjin, 300071, China

*Email: yuhua@nankai.edu.cn



Abstract

When a drinking straw is placed in a glass of carbonated drink, it can rise up, sometimes toppling over the edge of the glass. The motion process of straw in carbonated drink was studied by theoretical simulation and angular momentum theorem. The effects of the properties of containers, carbonated drinks and straws on whether the straws are toppling or not were studied by statistical methods.

1 Introduction

We often use straws to drink carbonated drinks. It is a very common phenomenon that straws float up and topple over in carbonated drinks. On the Internet, someone have once inserted a straw into a coke [1]. Because the bubbles in the coke stick to the straw, the straw keeps rising and eventually flips from the glass. In the observation, the longer the straw is, the easier it is to toppling, while the smaller the diameter of the container is, the easier it is to toppling. In addition, S.F. Jones et al. studied in detail the distribution of nucleation sites of carbon dioxide gas in carbonated beverages, the relationship between the volume of carbon dioxide bubbles and time, and the average attachment time of bubbles [2][3]. Nevertheless, the movement of straws floating and toppling in carbonated drinks has rarely been quantitative studied. Especially what factors lead to the final toppling of the straw has not been thoroughly researched. In this paper, the dynamics of straw in carbonated drinks was studied comprehensively based on the basic mechanical analysis. Then, the toppling conditions of different parameters in carbonated drinks were studied by statistical method.

2 Pre-experiment

Sprite is used as carbonated drinks and placed in beakers. Put different lengths of straws into drinks and observe their movements. When a straw is

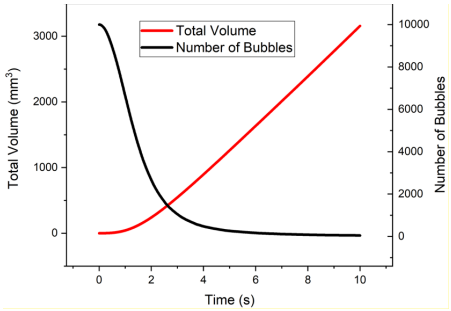


FIG. 1. The relationship between total volume of bubbles and time in carbonated drink.

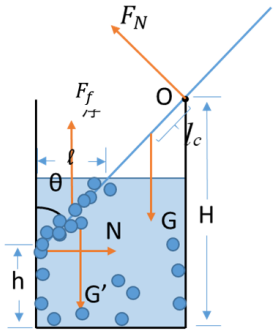


FIG. 2. Diagram of mechanical analysis of straws in carbonated drink.

put into a carbonated drink, many bubbles are produced on the straw. After a while, the straw began to float slowly until it reached the liquid level of the drink. When the straw is long, it is very easy to toppling over, but when the straw is short, the phenomenon of toppling is very difficult to occur or even not to toppling at all.

When water is used instead of carbonated drinks for the above experiments, the floatation and toppling of the straw will hardly occur unless the straw is very long. It can be seen that the bubbles in carbonated drinks cause the straw to float. And the length of the straw, and maybe some other factors, will eventually cause the straw floating on the surface of the carbonated drinks to eventually topple over.

3 Theory

According to the observation results of the pre-experiment, the motion of the straw in carbonated drinks can be divided into two stages, i.e. the floating and toppling of the straw. Three assumptions are proposed during the bubble generation stage. First, the bubbles are evenly attached to the surface of the straw [2]. Secondly, the bubble shape is approximately spherical [4]. Finally, there is a linear relationship between the total volume of bubbles and the generation time of bubbles [2][3]. For the last assumption, it can be expressed as

$$\frac{dV}{dt} = kl_0 \tag{1}$$

where V is the total volume of bubbles, t is the generation time of bubbles, l_0 is length of straw below liquid level and k is a constant determined by experimental measurement. To verify the rationality of the assumption, the results of numerical simulation by ChemLab and experimental measurements are shown in Figure 1. For a certain period of time, the total volume of bubbles increases linearly [2][3]. The numerical simulation results are in good agreement with the experimental observations.

More and more bubbles are attached to the surface of the straw, which makes the buoyancy of the straw increase continuously. At this time, the motion of the straw enters the first stage, that is, the floating of the straw. Basic mechanical analysis of straws in carbonated drinks is performed, as shown in Figure 2. Angular momentum theorem is used to analyze the process of straw floating up.

$$\frac{d\vec{J}}{dt} = \vec{M} \tag{2}$$

Where J is the angular momentum of the system and M is the total torque. At this stage, the influence of surface tension is neglected. Total torque can be divided into two parts: motivate torque and resistance torque. The motivate torque is buoyancy torque M_B generated by bubbles attached to straw. The resistance torques include gravity torque of straw M_G and their internal liquids M_{enc}, supporting torque of beaker to straw M_N, and fluid resistance torque of straw M_f. Only consider their quantity, therefore, the total torque M can be expressed as

$$M = M_B - M_N - M_G - M_{enc} - M_f \tag{3}$$

Among them, the expressions are as follows.

$$M_B = F_B\left(\frac{H-h}{\cos\theta} - \frac{l}{2}\right) \tag{4}$$

$$M_N = -N(H-h) \tag{5}$$

$$M_G = -Gl_c\sin\theta \tag{6}$$

and

$$M_f = c\omega\left(\frac{H-h}{\cos\theta}\frac{l}{\sin\theta} - \frac{l^2}{2\sin^2\theta}\right) \tag{7}$$

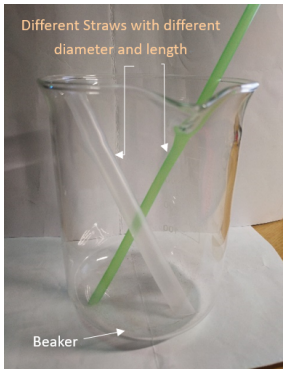


FIG. 3. Image of the experimental device for the straws in carbonated drink.

Where H is the height of the beaker, h is the distance between the contact point between the straw and the beaker wall and the bottom of the beaker, l is the distance between the contact point between the straw and the liquid surface and the beaker wall, l_c is the distance between the center of mass of the straw and the reference point O, θ is the angle between the straw and the beaker wall, G is the gravity of the straw, c is the laminar resistance coefficient, v is the relative velocity of straw and liquid and ω is angular velocity. The expression of the final angular momentum theorem is as follows:

$$\begin{aligned}\frac{dJ}{dt} = F_B\left(\frac{H-h}{\cos\theta} - \frac{l}{2}\right) - N(H-h) \\ - Gl_c\sin\theta - M_{enc} - M_f\end{aligned} \tag{8}$$

The expression gives a complete description of the floating process of straw in carbonated drink.

When the straw floats to the liquid level in carbonated drinks, the motion of the straw enters the second stage, i.e. toppling stage. As the straw has reached the liquid surface, buoyancy torque M_B produced by bubbles and fluid resistance torque of straw M_f produced by liquids disappear. The gravity torque M_G produced by the straw is redefined by a simple geometric relationship. The angular momentum equation of the system at the toppling stage becomes

$$\begin{aligned}\frac{dJ}{dt} = -N(H-h) \\ - G\left[(H-h)\tan\theta - \frac{L}{2}\sin\theta\right] - M_{enc}\end{aligned} \tag{9}$$

Because the floating and toppling of straw are a continuous process, the equations 8 and 9 are used to determine whether the straw is toppling or not. In the final toppling stage, we have to point out that, the surface tension can no longer be neglected, since the contact edge between the straw and the fluid surface is quite long. Through the analysis of the equations 8 and 9, we can know that, the straw would topple when the angular momentum of the straw is greater than zero after it leaves the liquid surface. That is to say, when the length of the straw, temperature and the height of the liquid becomes larger, the straw would topple over the edge of the beaker more easily. While on the other hand, when the height of the beaker, the diameter of the beaker and the straw becomes greater, the straw would tend to stay in stationary.

4 Experiment and discussion

The experimental device is shown in Figure 3. Sprite is placed in beaker to simulate carbonated drinks in life. Pipes with different lengths (5 cm, 7 cm, 8 cm, 10 cm) and internal diameters (1.6 mm, 2.0 mm, 2.4 mm, 2.9 mm, 3.4 mm, 3.8 mm, 4.0 mm, 4.3 mm) are used to investigate the motion of straw in carbonated drinks. Thermometers are used to record the temperature of the experiment. A high-speed camera is used to record the motion of the straw. Tracker software is used for data processing of video.

In the process of floating, considering the gradual disappearance of bubbles as the straw rises, the change of the total volume of bubbles with time can be expressed as

$$\frac{dV}{dt} = kl_0 - kt\frac{d}{dt}\left(\frac{h_0-h}{\cos\theta}\right) \tag{10}$$

To obtain this expression, we assume that:
1. The bubbles that contribute to the motion of the straw are meanly developed on the surface of the straw.
2. Assume the shape of the bubbles are sphere like.
3. The total volume of the bubbles has a linear relationship with time and the length of the straw in the water.

Combining equation 7 and using Mathematica for numerically simulation, the corresponding curves for describing the process of straw floating are shown in Figure 4. Meanwhile, the data points of the relationship between the time and the angle between the straw and the beaker wall are also marked in Figure 4. The two coincide well. This shows that the theoretical analysis of the floating stage of the straw can well describe the motion process of the straw.

For the experimental verification of whether the straw is toppling or not, based on the theoretical results, five parameters are selected, including the temperature of carbonated drink, the height of liquid surface in beaker, the length of straw, the inner

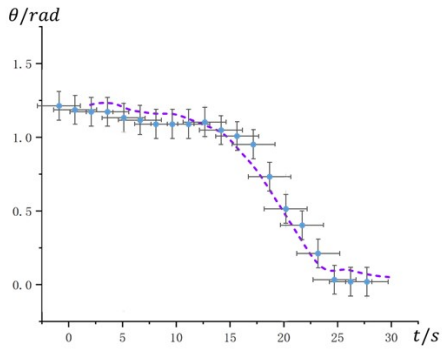


FIG. 4. The relationship between the time and the angle between the straw and the beaker wall.

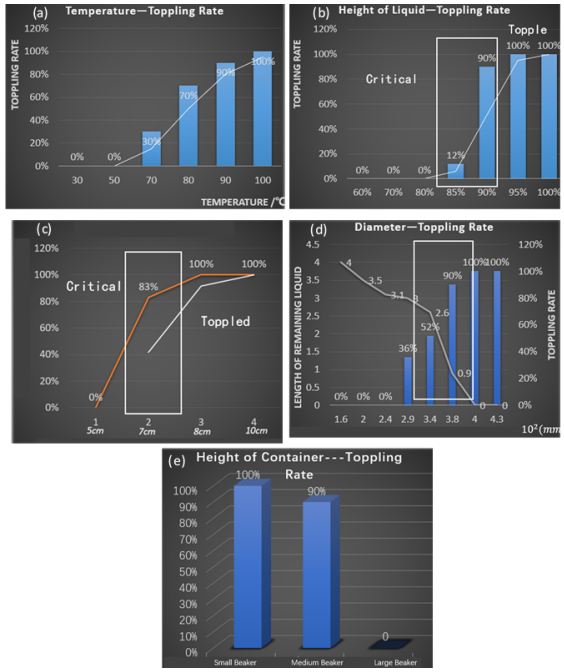


FIG. 5. (a) The relationship between the toppling rate of the straw and the temperature of carbonated drink, (b) the height of liquid surface in beaker, (c) the length of straw, (d) the inner diameter of straw and (e) the bottom area of beaker.

diameter of straw and the bottom area of beaker. Because the toppling judgment of straw is highly sensitive to the temperature of the experimental environment, statistical methods are used to make the experimental results more convincing. In the experiment, under the condition of controlling variables, the method of calculating the toppling rate of the straw was adopted to study the condition of the toppling of the straw. The experimental results are shown in Figure 5. For beaker with carbonated drink, the larger the beaker bottom area, the easier the straw to toppling. For carbonated drinks, the more the amount in the beaker and the higher the temperature, the greater the probability of the straw toppling. For the straw itself, the probability of toppling depends on the longer and thicker pipette. The experimental results are consistent with the theoretical results. The experimental results are not surprising, since more bubbles will be produced under a higher temperature, and when the temperature continue goes higher, we have to take into consideration of the impulse that moving bubbles exert to the straw. When the diameter of the beaker goes up, or, the length of the straw becomes longer, less resistance torque will be exerted to the straw from the beaker. And if the beaker carries more carbonated drink, floating torque will continue exerting to the straw until the straw reaches the top of the liquid surface.

However, we have to point it out that, since the carbonated drink is not the same in different bottle, a carbonated water production machine is suggested in a more precise experiment. By doing so, more parameters can be investigated including the concentration of carbon-dioxide. In our experiment, due to the limit of our experiment conditions, only the trend of how relative parameters affect the toppling result is verified.

5 Conclusion

Due to the increasing of straw buoyancy caused by the bubbles attached to the straw, the straw gradually floats to the liquid surface of the drink and then topples. The angular momentum equations based on force analysis can theoretically fully describe the motion process of the straw in carbonated drink. It is found that the floating and toppling of straw are a continuous process. The longer and thicker straws are easier to toppling. In addition, the larger volume of carbonated drink, the higher the temperature and the larger the bottom area of the container will also lead to the toppling of straw.

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