2015 Problem 9 : Hovercraft

What affects the low friction state

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Abstract

A simple model hovercraft is built using a CD and a balloon filled with air attached via a tube. Exiting air can lift the device making it float over a surface with low friction. The aim is to find the parameters which influence the time of this low friction state. This following article consists of two parts. In the first part, we measured the pressure variations inside the balloon and the effect of reusing the same balloon, hence we linked it to the chemical properties of rubber. Then we investigated several possible parameters such as the mass loaded on the disk, the size of exit hole, the floating height and the smoothness of the surface. In the second part, we built up a model for the behavior of the hovercraft. The final equation for the lift force links the parameters we found together, and it explains the mechanism underneath the disk.

Introduction

The key term here is ‘low friction state’. When the air is released, the device can move freely on the surface as it is not touching. We define that as the low friction state.

The pressure inside the balloon is larger than the atmospheric pressure, so air is pumped out of the balloon. It flows radially outward from the exit hole and is opposed by viscous forces. The force required to overcome the viscous forces results in a radial pressure gradient. The lifting force is the integral of excess pressure over the surface area of the disc. There is an additional lifting force arising from the change of momentum of the air underneath the hole, as shown in Figure.1.

EXPERIMENTS

Pressure inside the balloon

Using the water manometer shown in Figure 2, we found the pressure inside the balloon. From Figure 3, it is very easy to see that

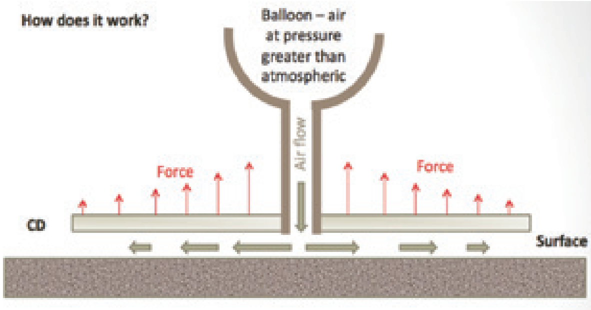

Fig.1. A cross-sectional outline detailing the physics behind our CD hovercraft.

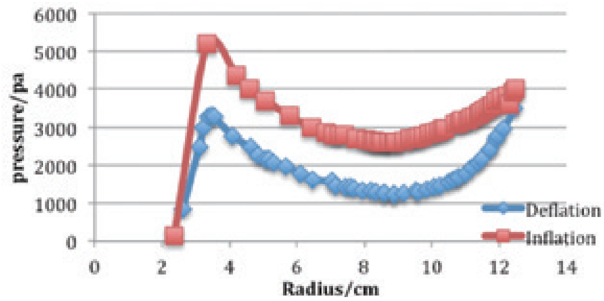

Fig.3. Pressure-radius graph of the balloon during inflation and deflation. There is a clear disparity between the pressure inside the balloon for a given radius during inflation and deflation.

the relationship between the excess pressure inside the balloon and the radius of the balloon is not a single-value function and there is a clear disparity between the pressure inside the balloon for a given radius during inflation and deflation.

However, the excess pressure is still very small compared to the atmospheric pressure (during deflation the average excess pressure for this balloon is ~2000 Pa, only 2% of the atmospheric pressure) - so we assume that the air inside the balloon has approximately constant density.

What causes this kind of trend? This is due to the chemical structure of the material of the balloon - rubber. Rubber does not obey Hooke's Law when it deforms. It's made up of long chain polymer molecules joined up in various places along the chain known as cross-linkages. In its natural state, the chains are twisted, so a great amount of force is needed to stretch them at the beginning and the pressure increases rapidly. Once they are pulled apart, the force needed to pull them further are reduced, however, they are still connected with each other until the force is big enough to break all the cross-linkages between chains. At this stage, you are actually stretching the molecules themselves.

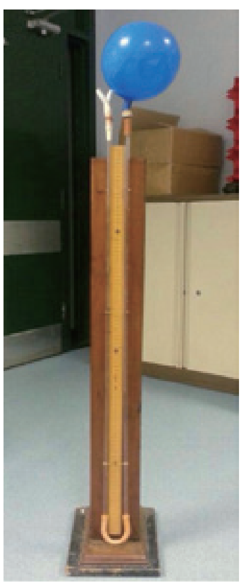

Fig.2. A water manometer is used to measure the excess pressure inside the balloon.

Reusing the same balloon

Due to this property of rubber, we also considered the effect of reusing the same balloon.

A fresh balloon was inflated to a given volume and we recorded the time taken to deflate completely. The experiment was repeated 30 times. As the recordings show in Figure 4, the deflation time increases during the first few tests then it gradually becomes constant,

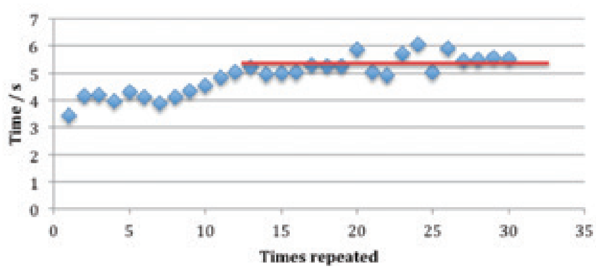

Fig.4. A fresh balloon was inflated to given volume and we recorded the time taken to deflate completely. The experiment was repeated 30 times.

Fig.5. We filmed the process of deflation, and then we analysed the video to determine the diameter of the balloon.

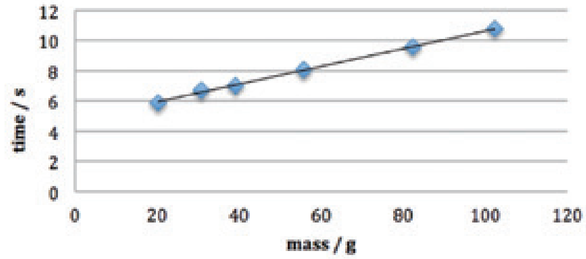

Fig.7. Mass-time graph of the balloon during deflation. The mass refers to the total mass of the hovercraft.

levelling out about after the 13th time. We then carried out the rest of the experiment using the same balloon.

Balloon deflation

We measured the rate of change of volume during deflation from a video clip. The volume is calculated by the average radius measured in different perpendicular dimensions shown in Figure 5.

The graph in Figure 6 shows that Q, the volume of air flow per second during the deflation of the balloon is constant. It is about $-9\times10^{-4} m^3/s$ - calculated from the gradient of the line in Figure 6.

Varying the loaded mass

We placed the masses evenly across the disc and measured the average time of the low friction state.

We can tell from the graph shown in Figure 7 that the time of the low friction state has a positive linear relationship with the loaded mass. In order for the disc to remain in low friction state for a longer period of time, the rate of air flow, Q, must have been lower. However, the air cushion is supporting a greater weight so the lift force must have increased. This means that a greater lift force is being generated from a smaller value of Q. This can only be possible if other factors, such as the floating height, have changed.

Different hole sizes

We also investigated if the exit hole size would affect the rate of air flowing out. By measuring the time taken to completely deflate a given balloon volume over a range of different exit hole sizes, we found that smaller holes takes shorter time. Air exits a smaller hole faster than a larger hole, as shown in Figure 8.

The floating height, h

In order to measure the floating height, we stacked sellotape repeatedly on the surface, lifted the hovercraft then let the hovercraft pass over it [2]. When the

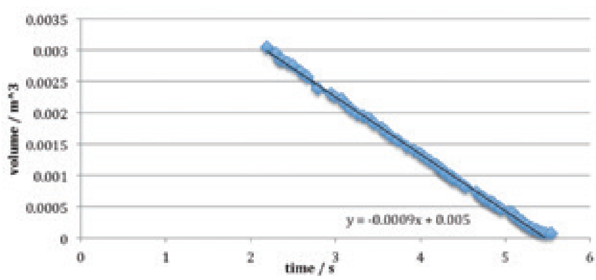

Fig.6. Volume-time graph of the balloon during the deflation.

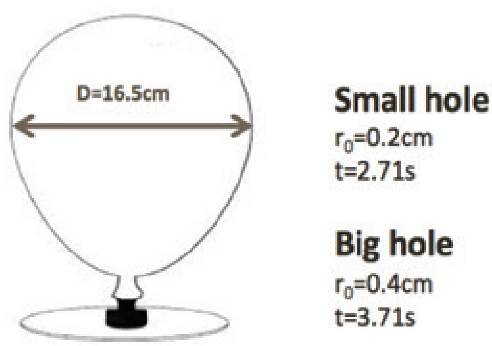

Fig.8. Different exit hole sizes were used.

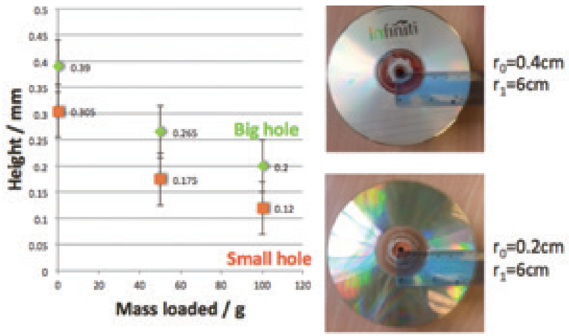

Fig.9. Floating height-mass loaded graph. Different sizes of holes were compared. r0 refers to the radius of the exit hole. r1 refers to the radius of the disc.

hovercraft couldn't pass over it, we knew that the floating height was equal to the thickness of the strip. The thickness of the strip was measured by a digital thickness gauge.

As shown in Figure 9, we measured the floating height on hovercrafts with different hole sizes and mass. We found that for a constant mass, the height for smaller holes is lower. As the mass loaded increases, the height (thickness of supporting air film) decreases regardless of the hole size.

Varying the surface

Three different surface conditions were chosen. As shown in Figure 10, the surface of the file is the smoothest. The surface of the table is slightly coarser, and the surface of the floor is the coarsest.

Time for the low friction state decreases as the roughness of the surface increases. We believe that this is because when the surface is rougher, the air escapes through the gaps on the surface and so the air flow is increased.

Fig.10. Three different surface conditions were chosen. File (the smoothest), table (slightly rougher) and the floor (the roughest).

Model of hovercraft

Based on the experiments we did, we are now trying to construct a mathematical model which links the parameters for the lift force of the whole device. In the following calculation, we take the uncertainties of h into account.

Initial calculation

For a hovercraft with: r_0 (radius of the exit hole) = 0.004 m, r_1 (radius of the disc)= 0.060 m and h = 0.0004 m (approx. for a hovercraft of mass about 20 g). The time it takes to completely deflate from the initial balloon diameter of 0.230 m is about 5.0 s. Assuming the rate of air flow Q is constant, this gives

$$Q = 4/3\pi\,(0.23/2)^3\,/5.0 = 1.27\times10^{-3}\,m^3s^{-1}$$

(which is consistent with our measurements).

We can calculate the flow velocity at radius r using the equation of continuity:

$$Q = 2\pi rvh \qquad (1)$$

where h = thickness of air layer. Therefore:

$$v = Q/2\pi rh \qquad (2)$$

So we have velocity of air at the centre hole

$v_0 = Q/2\pi r_0h \approx 83.3\ ms^{-1}$ and velocity at the edge

$v_1 = Q/2\pi r_1h \approx 5.56\ ms^{-1}$

Flow in the air layer

We used Reynolds number which states that

$$\text{Reynolds number } R_e = \text{inertial forces/viscous forces} = \rho vL/\eta \qquad (3)$$

The laminar flow is assumed if $R_e < 4000$.

ρ= air density – taken to be about 1 kg m^{-3}

v = air flow speed

L = characteristic length scale (taken to be h) ≈ 0.0004 m

η= absolute (dynamical) viscosity of air ≈ 1.85×10^{-5} Pa·s

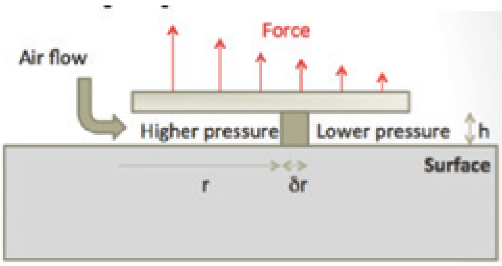

Fig.11. A cross-sectional outline showing the ring of air moving outwards from the exit hole.

From the equation, we get $R_e \approx 1800$ near the centre, and $R_e \approx 120$ near the edge. So here the laminar radial flow assumption seems to be reasonable.

Pressure variation with radius of the air ring underneath the disc

Considering a ring of air at radius r. It flows outwards at speed v against viscous forces as shown in Figure 11.

From the viscosity equation, we have

$$F/2\pi r\delta r = \eta\, dv/dz \qquad (4)$$

where dv/dz is the radial velocity gradient in vertical (z) direction as shown in Figure 12.

$$\frac{dv}{dz} = \frac{v}{h/2} = \frac{2v}{h} \qquad (5)$$

Note that a linear approximation is used for the velocity gradient between the two surfaces.

Because the air is flowing between two surfaces, we double the velocity gradient and substitute into Eq. (4) we get:

$$F = 8\pi r\eta v\delta r / h \qquad (6)$$

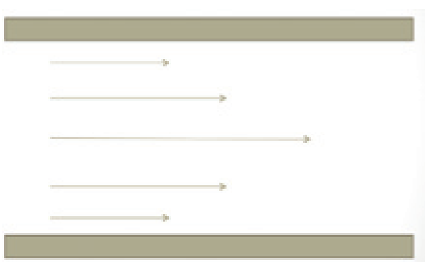

Fig.12. The velocity gradient

The viscous force creates a pressure gradient dp/dr. (p refers to excess pressure over atmospheric).

$$F = 2\pi rh\, \delta p = 8\pi r\eta v\delta r / h \qquad (7)$$

So $dp/dr = -4\eta v/h^2$ (negative because pressure falls to atmospheric at r = r1)

Substitute the v from Eq. (2)

$$dp/dr = -2\eta Q/\pi rh^3 \qquad (8)$$

The excess pressure from the atmospheric pressure is calculated by integrating from r1to a radius r, which is

$$p = 2\eta Q/\pi h^3 \ln(r1/r) \qquad (9)$$

Using the typical value for the blue balloon, gives a pressure at $r=r_0$ of

418 Pa (h=0.0004 m) 161 Pa (h=0.00055 m).

These values are much significantly lower than the excess pressure inside the inflated balloons. The reason for this is that there must be a pressure gradient down the balloon neck to accelerate the air to its initial speed at $r=r_0$.

Lift under the disc

Since the pressure change from r_0 to r_1 is very small compared to the atmospheric pressure, a linear approximation is used here to find the lift force.

$$F_{lift} = \int_{r_0}^{r_1} 2\pi r p\, dr \qquad (10)$$

Substituting for p and integrating by parts gives:

$$F_{\text{lift}} = \frac{2\eta Q r_1^2}{3h^3} \ln\left(\frac{r_1}{r_0}\right) = 0.607\text{N}(h=0.00055\text{m})\ 1.58\text{ N}(h=0.0004\text{ m})$$

This increases the static pressure under the disc and also decelerates the rate of the air as it moves outwards. After subtracting, we have

$$F_1 = \frac{2\eta Q r_1^2}{3h^3} \ln\left(\frac{r_1}{r_0}\right) - \frac{\rho Q^2}{4\pi h^2} \ln\left(\frac{r_1}{r_0}\right) =$$

0.004N (h=0.00055 m) 0.44 N (h=0.0004 m)

Lift under centre hole

Assuming the excess static pressure under the hole is equal to the pressure at r_0. This gives an additional lifting force:

$$F_2=\pi r_0^2 p_0 = 0.02\ \text{N}\ (h=0.0004\ \text{m})\ 0.008\ \text{N}\ (h=0.00055\ \text{m})$$

Forces due to change in momentum

The additional force arises when air at the centre changes its direction: initially the air is moving downwards with speed v0 but then it moves horizontally).

Force = change in momentum = $Q\,\rho\, v_0$

Substituting the typical value,

$$F_3=0.06\ \text{N}\ (h=0.00055\ \text{m})\sim 0.08\ \text{N}\ (h=0.0004\ \text{m})$$

Comparison of theory and experiment

Adding these three forces together, we now have an expression for the total lift force:

$$F_{total}=\frac{2\eta Q r_1^2}{3h^3}\ln\left(\frac{r_0}{r_1}\right)+\pi r_0^2 p_0+Q\,\rho\, v_0-\frac{\rho Q^2}{4\pi h^2}\ln\left(\frac{r_1}{r_0}\right) \quad (11)$$

For the hovercraft we used here, the total mass of hovercraft=20.2 g therefore the weight of hovercraft=0.198 N. Putting the data we got, the total lift force=$F_1+F_2+F_3$ gives a range of 0.1 N to 0.5 N which is consistent to the weight of the hovercraft.

Conclusion

In the article, we found out that the time for the low friction state depends on Q, h , r_0 (radius of the hole), Initial volume and the roughness of the surface. (We also expect it to depend upon the radius of the disc, r_1.) Moreover, we not only investigated the factors which affect on the time of low friction state, we also think it is very important to understand the physics nature of the low friction state. We produced a model of the lift force showing it is made up of four components which are due to:

1. The pressure under the disc resulting from motion against viscous forces in the air
2. The static pressure under the central hole
3. The dynamic pressure under the central hole due to change of momentum of the air
4. A negative contribution because the air is decelerated as it spreads out radially.

The largest source of uncertainty would be the measurement of the floating height (which appears to powers 3 and 2 in the terms above). This leads to a large overall uncertainty in theoretical lift but a range of values consistent with the actual weight of the hovercraft used.

Glossary

r_0- the radius of the exit hole

r_1- the radius of the disc

Q- the volume of air flow per second

ρ= air density – taken to be about 1 kg m^{-3}

v = air flow speed

L = characteristic length scale (taken to be h) ≈ 0.0004 m

η= absolute (dynamical) viscosity of air ≈ 1.85×10^{-5} Pas

p_0- pressure at the centre hole

h- floating height

Acknowledge

I would like to thank Mr. Steve Adams and Mr. John Balcombe for helping me with the experiment.

References

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[2] Method used to measure the floating height C. de Izarra and G. de Izarra. Stokes equation in a toy CD hovercraft. Eur. J. Phys. 32, 89-99 (2011)