2010 Problem 12 : Breaking Spaghetti

Find the conditions under which dry spaghetti falling on a hard floor does not break.

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Chong Wang
Department of physics,
Nanjing University

Chengping Zhang
Department of physics,
Nanjing University

Abstract

The mechanism of breaking spaghetti when it falls on a hard floor is investigated. Landing velocity and angle influence are key parameters to determine how it ruptures. We classify breaking mechanisms into three types. In each type, qualitative analysis is given and is compared to experiment results. Other parameters such as humidity which can also influence rupture are also investigated.

Keywords

spaghetti, rupture mechanism, landing angle, landing velocity

Introduction

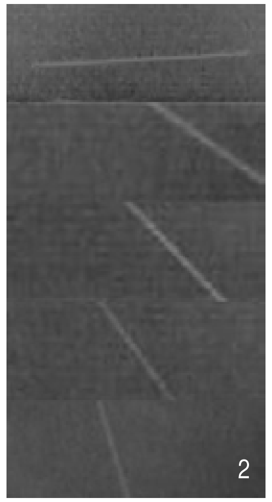
This is a solution to the 4th problem of IYPT 2011. The problem is "Find the conditions under which dry spaghetti falling on a hard floor does not break."

How spaghetti falling on a hard floor breaks is not as a simple problem as it looks. Dynamic models of elastic rod was studied by Liu Yanzhu etc. and simulation is feasible according to these models (see reference [1-3]), but rupture mechanism is not well understood before. We research experimentally and explain the results by introducing three mechanisms.

Experimental observation and rupture mechanism

To control the landing velocity (the velocity just before spaghetti reaches floor), we fall spaghetti from different steps of stairs, as is shown in fig.1. The initial angle is fixed before releasing. As a result, the angular velocity of spaghetti is very small when it hits the floor. But to be more precise, the actual landing velocity and landing angle (the angle between spaghetti and floor just before spaghetti reaches floor) are obtained by analyzing the video clips of Casio EX-FC150 high speed camera (1000fpm) [fig.2]. We record the landing velocity and landing angle of each video clip and state of the spaghetti after landing from hundreds of experiments, see fig.3.

FIGURE 1-experiment site



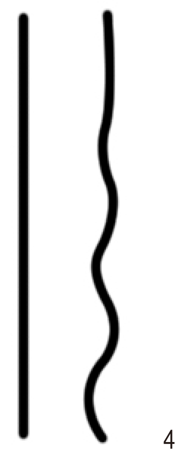


FIGURE 2-snapshots of videos of different landing angles
FIGURE 4-buckling rupture

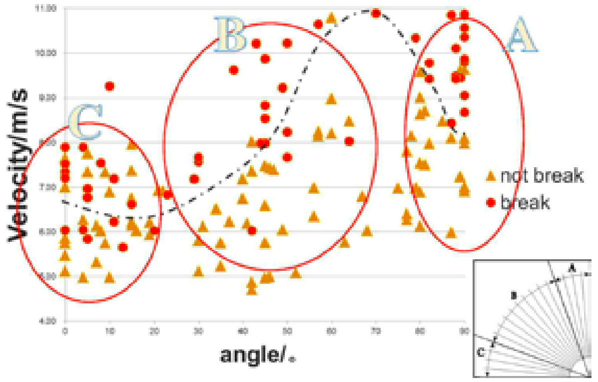


FIGURE 3-experiment results of angle and velocity. 'A' stands for buckling rupture; 'B' stands for curving rupture ;'C' stands for random rupture.(Inset: division of rupture mechanism by landing angle. 'A' stands for buckling type; 'B' stands for curving type ;'C' stands for random type.)

In fig.3, round dots indicate cases in which the spaghetti breaks and triangle dots indicate cases in which the spaghetti does not break. We can easily come to a conclusion from the diagram: when landing angle is bigger than about 70°, the probability of rupture decreases with landing angle; when landing angle is bigger than about 20° and smaller than about 70°, the probability of rupture increases with landing angle; when landing angle is smaller than about 20°the probability of rupture slightly decreases with landing angle. Notice that while clear border can be seen in areas labeled A and B, there is no clear border in C. This will be explained later.

Now, we can classify rupture mechanism into three types according to the landing conditions in areas A, B and C in fig.3.

A: Buckling rupture

Buckling type works when spaghetti hits the floor at a large angle (70°~90°). Rupture happens when one end of it hits the floor and bears a great force from the floor (keeping in touch with the floor).

In fig.4, spaghetti hits the floor at 90°(left). Soon afterwards, great longitudinal pressure appears in the spaghetti and causes instability. Due to the instability, lateral displacement appears (right). Lateral displacement is periodic (wave shaped, not oscillating), and moving outwards from the central axis. At last, critical curvature is reached and rupture happens.

To show how the 'amplitude' of the 'wave' increases, we give theoretical analysis here.

When spaghetti hits the floor vertically, a longitudinal wave appears. The wave travels at the speed of sound and gives rise to great pressure behind. Great longitudinal pressure leads to lateral wave-like displacement and 'wavelength' is calculated as (see reference [4]):

$$F_0 = Sv\sqrt{E\heartsuit} \qquad (1)$$

$$\infty= (\frac{\vartheta}{2})^{1.5} d^2 \sqrt{\frac{1}{Sv}\sqrt{\frac{E}{\heartsuit}}} \qquad (2)$$

where $\boldsymbol{F_0}$ stands for longitudinal pressure; $\boldsymbol{d}$ stands for diameter of the spaghetti; $\boldsymbol{S}$ stands for cross section area; $\boldsymbol{v}$ stands for landing velocity; $\boldsymbol{E}$ stands for Young's modulus; $\boldsymbol{\rho}$ stands for density scale.

If the speed is greater, λ will become smaller, which leads to a relatively bigger curvature and the spaghetti will be easier to break.

Also, if the landing angle is not big enough, the pressure will become lower, which will protect the spaghetti from rupture. That is convinced in fig. 3(A).

According to our parameters, E=3Gpa and ρ=1440kg/m^3, take the average landing velocity v=9m/s, the wavelength is around 4.8 cm, the maximum curvature is estimated at $\frac{\lambda}{4}$, i.e. 1.2 cm, where the spaghetti will break. Average experiment data shows spaghetti breaks at 1.4 cm, relatively consistent with theory.

B: Curving rupture

The landing angle of this type is 20°~70°. In this case, spaghetti does not break when just one end of it hits the floor——part of the spaghetti lie on the floor while the rest is squeezing down to cause a bending. Different parts hitting the floor on different time is crucial to cause great curvature. We will illustrate a typical curving rupture [fig.5]. The left part of the picture shows an approximate process of landing, and the right part shows the curvature change in the bending part before rupture. At first, the spaghetti hits the floor. Then, a flexural wave appears [fig.5 (b)]. Next, right part of the spaghetti adheres on the floor and the maximum curvature increases. Finally, rupture happens as curvature exceeds its limit [fig.5 (d)].

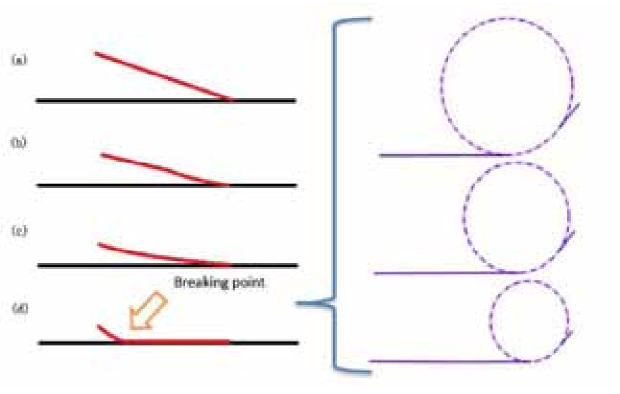


FIGURE 5- a typical form of curving rupture

To get further analysis, we are going to introduce rupture mechanism from energy perspective. In fig. 5, a typical rupture is displayed. Shortly after spaghetti hits the floor, potential energy reaches maximum [fig.5(c)]. However, in most cases, it is still not big enough to break the spaghetti, so the energy has to be concentrated on a small section. Spaghetti breaks when critical curvature is reached.

From the analysis above, we found three factors important:

1. *The initial energy (gravitational potential energy)*
2. *The ratio that kinetic energy transforms into (elastic) potential energy.*
3. *The speed of concentrating (elastic) potential energy. (Experiments show during the process the part of spaghetti in contact with floor become relatively straight and elastic energy is concentrated in those parts which are still in the air ,mostly in breaking point.[fig.5(d)] This process needs time, so in order to describe it, a velocity is introduced.)*

It is worthwhile to notice that factor 3 is very important. The whole process is like the oscillation of a rod. Kinetic energy and elastic potential energy take turns to transform into one another. If the speed of concentrating potential energy is not big enough, it will transform back to kinetic energy.

Greater velocity contributes to factor 1 and Greater angle contributes to factor 2 but impedes factor 3. Experiments show factor 3 matters more than factor 2. As a result, higher velocity and smaller angle contributes to rupture. This is also convinced in fig.3 (B).

This description is somehow oversimplified. As a matter of fact, there are many different forms of curving rupture.

C: Random rupture

In this case, the landing angle is small (0°~20°). The mechanism is similar to curving rupture but more complicated. Angular velocity (even very small), smoothness of floor and other parameters matters a lot in rupture. Due to small the landing angle, spaghetti may not undergo overstress at first hitting, and may bounce at both ends before come to a rest. Rupture depends on many factors and randomness plays an important role. The scattered dots on fig.3 area C shows such randomness. There are many forms of rupture in this case and no typical one can be found. However, great velocity still contributes to higher rupture probability.

Other parameters

Length of spaghetti

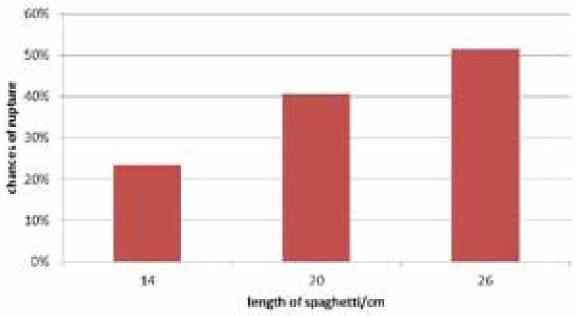


FIGURE 6-experiment results of length of spaghetti

We also fall spaghetti of different lengths from a certain height. Fig. 6 shows results of experiments in length of spaghetti. In this experiment, we fell spaghetti of different lengths from a certain height horizontally and calculated the probability. It can be easily concluded that rupture probability increases with length of spaghetti.

It is noteworthy to point out that experiments were done in random type. Longer spaghetti has more original energy, which increases the chance of rupture.

Floor and friction

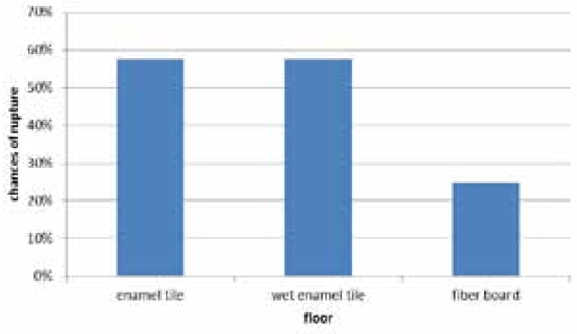


FIGURE 7- experiment results of floor and friction

The influence of floor and friction is also investigated. Fig. 7 shows results of experiments in different kinds of floor and friction. Again, spaghetti is placed horizontally before it falls. From this diagram, we can see softer floor "protects spaghetti" (compare

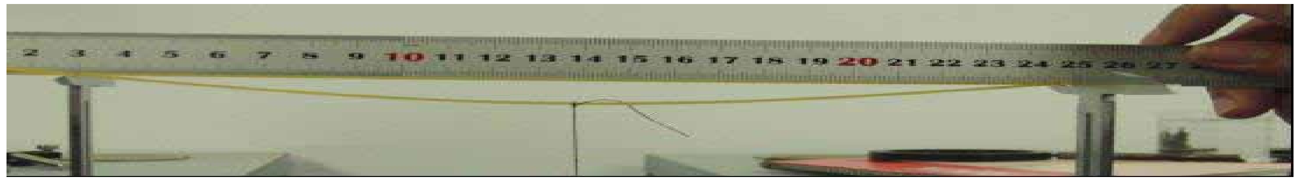


FIGURE 8- measure of Young's modulus and maximum deflection

enamel tile and fiber board), and friction does not matter a lot (compare enamel tile and wet enamel tile). Actually, because water can "protect spaghetti", friction may do have some influence, but it seems not matter a lot.

Humidity

Humidity also has an influence on rupture. We suppose it is because humidity influences the property of spaghetti and measured Young's modulus of spaghetti and maximum deflection of spaghetti.

As is shown in fig. 8, we put weights in the middle of spaghetti. Critical deflection is the deflection just before rupture. This is measured by high speed camera.

Young's modulus can be calculated by the following formula (see reference [5]):

$$E\ 6\ \frac{mgL^3}{12l\vartheta R^4} \tag{3}$$

where $\boldsymbol{L}$ is the distance between two knife-edges and $\boldsymbol{l}$ is deflection.

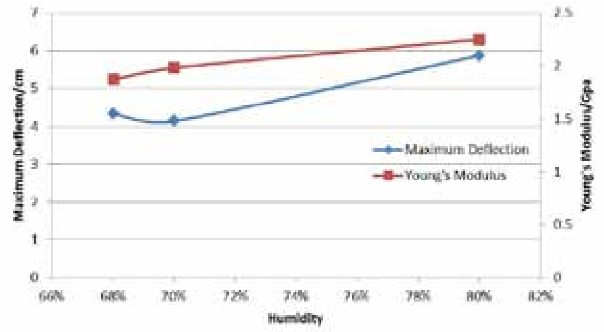


FIGURE 9-experiments of humidity

In fig.9, the relationship of surrounding humidity and rupture chances is shown. Spaghetti was placed in surroundings long enough. The longitudinal coordinates are Young's modulus and Maximum deflection. Young's modulus indicates whether spaghetti is prone to bend and Maximum deflection indicates the critic curvature of rupture. Now that both Young's modulus and Maximum deflection is positively related to humidity, rupture probability falls with humidity. A possible explanation is that water molecules in spaghetti can changes the property of spaghetti.

Conclusion

Many factors influence the rupture of spaghetti, among which landing velocity and landing angle are most complicated. Landing velocity and landing angle affect the rupture differently in different rupture mechanics, which are determined by landing angle. Factors such as length of spaghetti, hardness of floor, friction, and humidity have monotonic influence on this process.

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