2015 Problem 13 : Magnetic Pendulum

Experimental Study on The Phase Locking of Magnetic Pendulum

Minghui Xu

Hang Li

Zaiqiao Bai

Department of Physics, Beijing Normal University, Beijing, China

peter_xumh@163.com

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Abstract

Magnetic pendulum periodically driven by a coil can be in certain conditions phase-locked with the driving signal and lead to stable oscillation. We construct the experimental model first and do numerical simulations according to the differential equation. We study the transient process of the phase locking and measure the amplitude and phase of the pendulum in various controlling conditions. Based on the nonlinearity of pendulum and the phase-dependent energy transfer between the pendulum and driving coil, a negative feedback mechanism is identified in the system, from which all the experimental results can be quantitatively explained.

Experiment setup

The magnetic pendulum consists of a rigid arm whose diameter is 2.40mm on a pivot (supper pulley offered by PASCO Company). It moves in a two-dimensional plane in low friction, with a small permanent magnet (dimension d=1.30cm, length l=2.40cm) fixed on its free end. A 500 turns electromagnet with outer diameter of 66 mm, inner diameter of 60 mm, and length of 20 mm (self-inductance L=19.17mH and resistance R=22.78Ω) is installed 0.60cm below the lowest point of the pendulum's motion. Connect the electromagnet with alternating power whose frequency can be varied from zero to thirty hertz. The angular region where the permanent magnet was influenced by the magnetic field of the electromagnet was expressed by 'α'. When we release the magnetic pendulum at any random starting position, the pendulum' motion evolves into a stable oscillation mode[1]. What's more, it was found that if we give some disturbance to this stable system, the magnetic pendulum can be stable at another amplitude.

To confirm whether the pendulum was stable, we used the Tracker to track the permanent magnet and found that it remained at the same amplitude after about fifteen minutes.

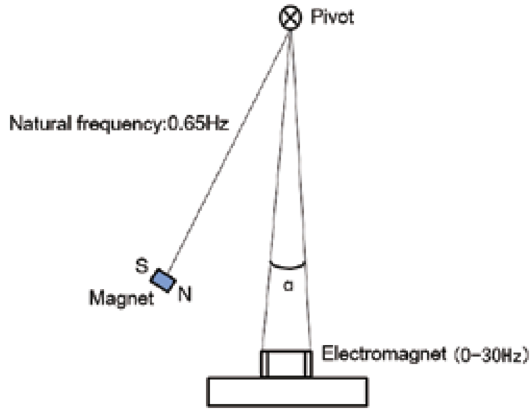

Fig.1. Diagram of magnetic pendulum. The moment of inertia of the pendulum is 0.064/kg·m^2. Natural frequency is about 0.65Hz. Magnetic polo of permanent magnet is marked in Fig.1.

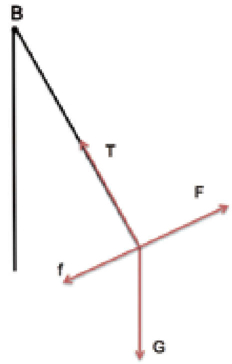

Fig.2. Force analysis diagram. The pendulum is forced by gravity, magnetic force, friction and tension.

Theory

Formulate the differential equation to describe its motion.

The equation of law rotation with respect to B point is

$$\Sigma M = I\alpha \quad (1)$$

where ΣM is the sum of torque. Letter I is the moment of inertia. α is the Angular acceleration of magnetic pendulum.

Thus we get

$$\Sigma \vec{M} = \vec{M_F} + \vec{M_D} + \vec{M_{mg}} \quad (2)$$

$\vec{M_F}$ is the moment of magnetic force. $\vec{M_D}$ is the moment of friction. $\vec{M_{mg}}$ is the moment of gravity.

According to equation (1) and equation (2), we can get

$$\ddot{\theta} + \frac{g\sin\theta}{r} + \frac{f}{mr} = \frac{cK(\theta)\cos(\omega t)}{mr} \quad (3)$$

where θ is the deflection angle of the magnetic pendulum. m is the mass of the permanent magnet. g is the gravitational field strength. f is the friction from the pivot. c is the peak of the magnetic force. Function $K(\theta)$ describes the change in the interaction zone. But we should note that this differential equation according to mechanics is of a type that cannot be solved easily by mathematical tools. So we wondered to do some numerical simulations.

Numerical simulation

Parameter analysis:

1. Peak of magnetic force

Connect the electromagnet to direct current. And move the electromagnet laterally with the force of pendulum being in balanced. Then we can calculate the maximum of magnetic force.

2. Function *K(θ)*

As we know, magnetic force has a short range. Once escaping from the interaction zone, the magnetic

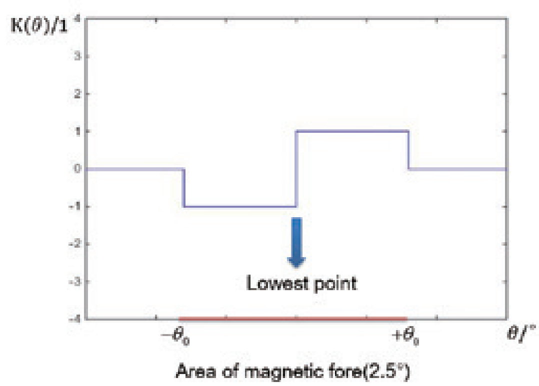

Fig.3. Diagram of function K(θ)

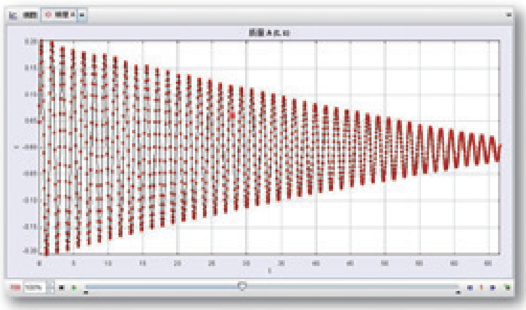

Fig.4. Decayed oscillation without kick-up by electromagnet.

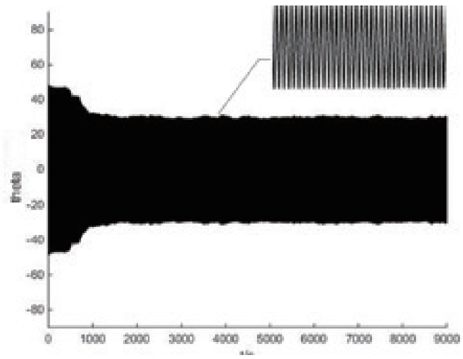

Fig.5. Evolution of oscillation. Enlarge the photo we can see the stable amplitude.

force will soon decay until disappear. Besides, we have premise that the direction of the magnetic force is adverse on both sides of the lowest point of motion so that the pendulum can get the maximum energy from the interaction zone.

Take function K(θ) as the simply format below, K(θ) is a dimensionless quantity.

$$K(\theta) = \begin{cases} 1, & 0 < \theta < \theta_0 \\ -1, & -\theta_0 < \theta < 0 \end{cases} \quad (4)$$

3. Friction

We found that the fiction is mostly produced from the pivot in our experiment.

Fig.4 shows that Amplitude decaying without kick-up by electromagnetic is almost linear as is shown by the Tracker, thus we calculate the friction using energy conservation laws.

$$2f(A_n + A_{n-1}) = \frac{1}{2} k (A_n^2 - A_{n-1}^2) \quad (5)$$

where f is frictional force which is a constant with angle.Respect to fitting result of k, we get

$$f = \frac{1}{4} k \Delta A = \frac{1}{2} m\omega^2 \Delta A \quad (6)$$

Figuring out the parameters unknown, we do the simulation with equation (3)

Numerical simulation results are as follows:

The pendulum is stationary after fifteen minutes. If we enlarge Fig.5 in Matlab, we can see that the pendulum remains at the same amplitude all the time.

Improved experiment

We improved the experiment by adding the photocell and laser opposite to each other. Photo cell can produce electrical energy with incoming laser beam. When the pendulum hits its lowest point and pendulum rod covers the photocell from laser beam, direct current going through the photocell will make a sudden change.

Experiment shows that $f1$ (the frequency of magnetic field) is always the integral multiple of $f2$ (frequency of pendulum). In Fig.7, we find that the voltage in coil undergo ten cycle between two photocurrent pulse. That is, when$f1$ = 12.756Hz, $f2$=0.683Hz. Because the system exhibits a good mirror symmetry, this kind of phase locking which is symmetrical is not hard to seek.

Blue curve stands for alternating current in electromagnet, and the red one represents the direct current in the photocell. We can see that the time when

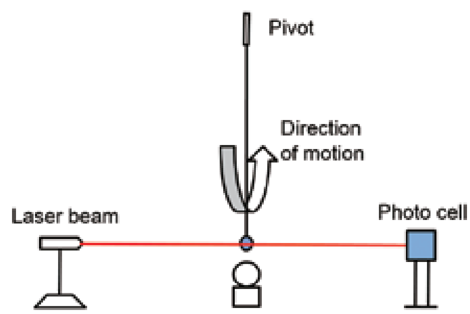

Fig.6. Improved experiment by adding laser and photo cell.

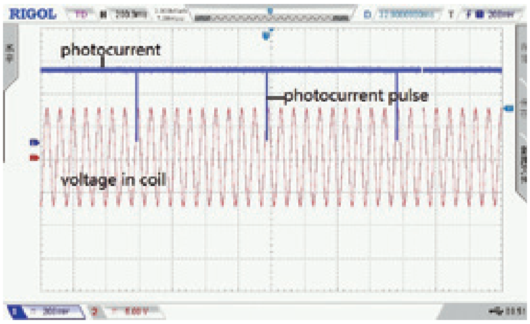

Fig.7. The frequency relationship between photocurrent and voltage in coil.

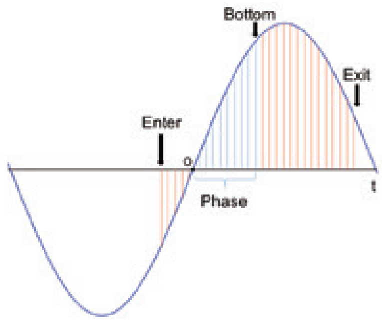

Fig.9. Further Analysis of energy and phase based on Fig.8

pendulum gets its lowest point was recorded in this way.

One premise: In our experiment, we found the cycle and amplitude of Magnetic pendulum shows a positive correlation. That is consistent with undamped simple pendulum whose relation between amplitude and cycle is

$$\frac{T}{T_0}=\frac{1}{\pi}\int_0^{\pi}\frac{d\varphi}{\sqrt{1-\sin^2\frac{\theta_0}{2}\sin^2\varphi}}\approx 1+\frac{\theta_0^2}{16}+\cdots$$

Fig.9 shows how the permanent magnet entries into the interaction zone and exists the interaction zone. We define a period of time starting from O point as Phase, which is colored blue. Red zone stands for obtaining energy and blue one stands for losing energy. It's evident that if we decrease the Phase, blue zone will be smaller. It's worth mentioning that if the magnet

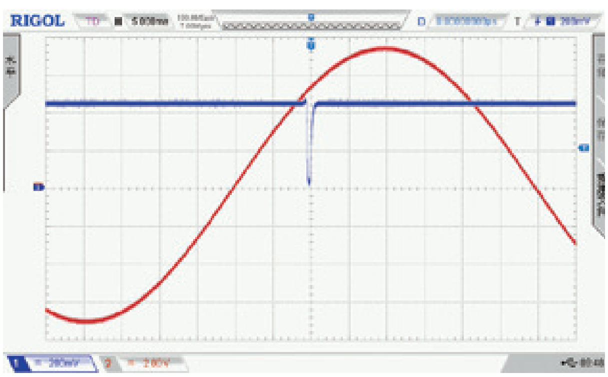

Fig.8. Voltage and photocurrent pulse curve captured from oscilloscope.

hits the bottom at O point, it can obtain energy utmost. That's why we find that the sudden change always occur in this area (Phase from 0 to $\pi/2$). Because the pendulum gets more energy in interaction zone.

We record a period of time during when the pendulum is going to be stable. Fig.10 shows that phase φ fluctuate over time, and finally gets to be stable. We found that the phase can be described properly by a damped oscillation function:

$$\varphi = a + b\cos(\omega t+\phi)\ \exp(-ct)$$

Using the function below to fit raw data, we set $f1=25.51Hz$ and various voltage to get Tab.1. It shows decay index is not a monotonic function of U.

Tab.1. Radian frequency **ω** and decay index c change with voltage in coil U (V)

U(V)	ω	decay index c
20.0	0.5990	0.0132
15.0	0.4550	0.0230
10.0	0.3327	0.0174

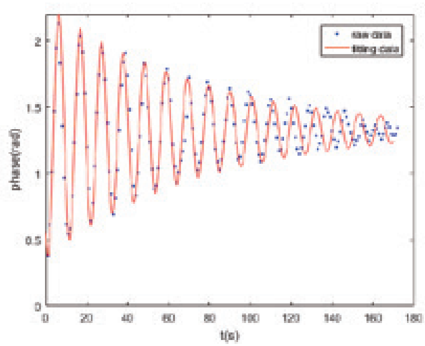

Fig.10. The relationship between phase and time.

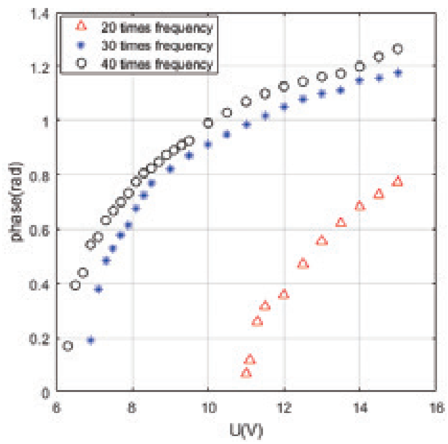

Fig.12. if f1=nf2,n=20,30,4, phase will be locked in different value by changing U (V). The explanation for phase locking is still tenable, since Phase and U shows positive correlation while Phase and frequency shows positive correlation.

To get more details about how the pendulum adjust itself to obtain energy to maintain its stationary state, we try to change the frequency and voltage step by step.

Firstly, we give some disturbance to the voltage in coil. The pendulum comes back to stable state after several cycles of adjustment.

Fig.11 U=15.0V in (a), when the pendulum is stable, change the voltage to U=11.1V step by step to get (b).

Comparing the above two graphs, the phase of Fig.11 (b) is lower than that in Fig.11 (a). Based on the state which shows in Fig.11 (a), I turn down the frequency or voltage, Phase will be lower like Fig.11 (b). Since the frequency and amplitude of Magnetic pendulum shows a negative correlation. If I turn down the frequency, the pendulum will have higher amplitude and moves faster in the interaction zone. In this case, it has to cut down Phase to obtain more energy in a very short period of time. If voltage is turned down, the pendulum will get lower energy the next time when it escapes from the interaction zone. So the pendulum reaches the bottom earlier to obtain more energy. That is, the phase will increase.

Result

We do some research on the phase locking mechanics of magnetic pendulum. The nonlinearity of pendulum is necessary. For the nonlinearity offers opportunity for pendulum to satisfy frequency constraint. Another necessary condition is the energy which is obtained from driving system is linked with phase. So, the pendulum can get energy conservation by adjusting phase to form stable oscillations. Actually, magnetic pendulum shows us a common mechanics called phase locking which can be found in many physical problems [2][3].

Acknowledgement

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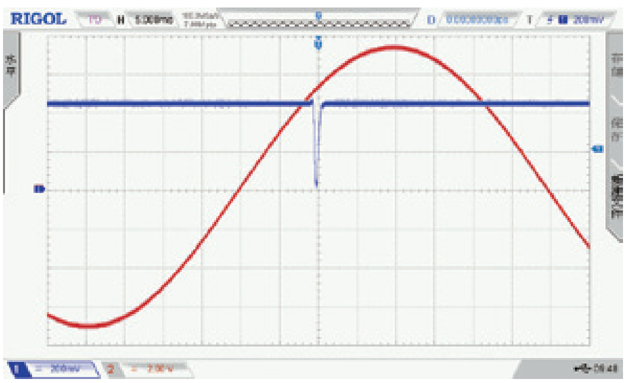

Fig.11 (a)

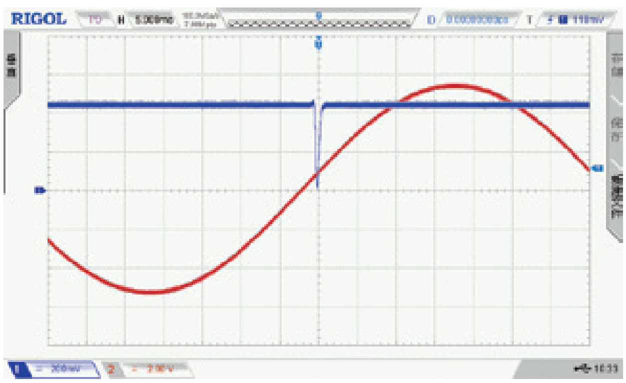

Fig.11 (b)

References

[1] Wikipedia, Doubochinski's pendulum

[2]Huygens C. Christiaan Huygens' the pendulum clock, or, Geometrical demonstrations concerning the motion of pendulum as applied to clocks [M]. Iowa State Pr, 1986.

[3]E. Fermi, Phys. Rev. 75, 1169 (1949).