2012 Problem 1 :Gaussian Cannon

When Collides With An Additional Steel Ball, The Final Ball Of A Line Of Steel Balls That Are Stuck To A Strong Magnet Will Shoot Away At Higher Speed.

Li Ni, Jiang Shuhan
Chen Letian
Liu Lisa

The School of Physics, Nankai University, Tianjin, China

1090832232@qq.com

Abstract

When a steel ball collides with a line of steel balls that are stuck to a strong magnet, the final steel ball of the line will be ejected at high speed like a bullet, even when the first ball have a rather low velocity. The theoretical analysis shows that the mass and initial velocity of the incoming ball, the susceptibility and the location of the magnetic ball are key factors that affect the output velocity. We find that no steel ball on the left hand side of the magnet is the best solution and it is supported by the experiment, but the number of steel balls on the right hand side depends on the properties of the system and it can be find by our experiment. In this paper, the magnetic scalar potential is the first time used to research the Magnetic cannon.

Introduction

Figure 1 shows the magnetic cannon system we studied. The radiuses of the magnetic ball and several steel balls are the same. During the process of the incoming ball approaching the magnet, it is being attracted, accelerating and finally stick to the magnetic. While the attraction between the magnet and the last ball is not that strong, according to the conservation of momentum, the last ball can get a

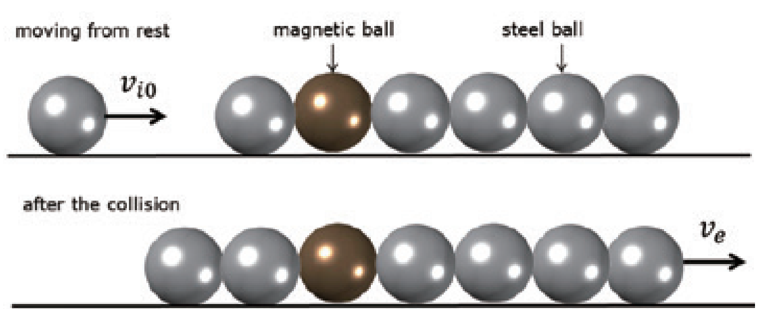

Figure 1. A simple magnetic cannon consists of a magnetic ball and several steel balls.

higher velocity after collision. We investigated the key reasons that affected the output velocity from theory analysis and experiments.

Theoretical analysis

Assuming the motion of the incoming ball is pure roll and the distance between the incoming ball and the other balls of the line is long enough that their interaction can be ignored in the beginning. The initial mechanical energy of the incoming ball is

$$E_{i0}=\frac{1}{2}mv_{i0}^2+\frac{I_i}{2R^2}v_{i0}^2+V(\infty)$$

Where m is the mass of the incoming ball, R is the radius of the incoming ball, v_{i0} is its initial velocity, $I_i=\frac{2mR^2}{5}$ is its moment of inertia an axis through its center of mass, and V is the potential energy of the incoming ball.

$$V=\frac{\mu}{2}\iiint_{V_0}(\nabla\varphi(r))^2\,dx^3 \qquad (1)$$

Where μ is magnetic permeability, V_0 is the volume of the incoming ball, and $\varphi(\vec{r})$ is the magnetic scalar potential of the incoming ball. The energy of the incoming ball at the moment before collision is

$$E_i=\frac{1}{2}m_iv_i^2+\frac{I_i}{2R^2}v_i^2+V(2n_iR+2R)$$

Where n_i is the number of steels balls on the left of the magnetic ball. According to the law of conservation of energy, the increase in kinetic energy is equal to the decrease in potential energy. And it leads to

$$\frac{7}{10}m_iv_i^2-\frac{7}{10}m_iv_{i0}^2=V(\infty)-V(2n_iR+2R)$$

During the collision, the rotational kinetic energy of the incoming ball will be completely lost because of the friction, and there is also some loss in the translational energy during collision. According to the law of conservation of energy, the velocity of the last ball should be

$$v_e=\sqrt{v_{i0}^2+\frac{10}{7m}|V(2n_iR+2R)|-\frac{\Delta E}{m}} \qquad (2)$$

Where ΔE is the translational energy loss during collision, which is related to the number of steel balls and the coefficient of restitution. It is obviously in equation (2) that the higher the initial velocity, the smaller the mass, the higher the output velocity of the last ball. The number of steel balls is also a key reason that affects the output velocity. If there are too many steel balls, more kinetic energy will be lost during collision process and the magnetic force on the last several balls will be so weak that more than one steel ball will shoot away. But if there is only one steel ball, the magnetic force may be strong enough to stop the steel ball from shooting away. Therefore finding the best number in experiment is necessary.

As the increase in kinetic energy comes from the decrease in potential energy, now we still need to calculate $V(2n_iR+2R)$. On the right hand side of the magnetic ball, those steel balls will be magnetized and produce induced additional magnetic field. It is possible to develop an expression in terms of the additional induction field which is produced by magnetization of those steel balls. The magnetic scalar potential of a lone magnetic ball with radius R and magnetization vector $\vec{M}$, equal to

$$\varphi(r)=\frac{R^3}{3}\frac{\vec{M}\cdot\vec{r}}{r^3} \qquad (r>R)$$

Since the motion of those balls were limited to the x-axis direction, $\vec{M}\times\vec{r}=0$, φ can be rewrite in this direction

$$\varphi(x)=\frac{m}{4\pi x^2} \qquad (x>R)$$

Where m is the magnetic moment of the magnetic ball, $m=\frac{4\pi R^3}{3}\vec{M}$. Assuming that every steel ball that remains at rest is influenced only by the magnetic field of balls on the left hand side of itself, $\varphi(x)$ can be rewritten in terms of the induced magnetic field.

$$\varphi_n(x) = \frac{m}{4\pi x^2} + \sum_{l=1}^{n} \frac{m_l}{4\pi(x - 2lR)^2}$$

$$m_l = \chi\left(\sum_{j=1}^{l-1} H_j + H_0\right)$$

Where n is the number of steel balls on the right hand side of the magnetic ball, l is the ordinal number of those steel balls, χ is the susceptibility, m_0 and H_0 are the magnetic moment and magnetic field strength of the magnetic ball, respectively. Since H_j is the partial derivative of $\varphi_j(x)$ for x, a recursion formula of $\varphi_n(x)$ is derived as below.

$$\varphi_{n+1}(x) = \varphi_n(x) + \frac{\chi \frac{\partial \varphi_n(x)}{\partial x}}{4\pi(x - 2(n+1)R)^2} \quad (3)$$

The scalar magnetic potential decreases with the increase of distance x. The additional induced magnetic field cannot offset the decrease. For this reason, it is the best solution that put no steel ball on the left hand side of the magnetic ball to get the highest velocity.

Through the theory analysis, we find that the numbers of balls on both sides of the magnetic ball, the initial velocity and mass of the incoming ball, the susceptibility are the key reasons affect the output velocity of the last ball. Higher the initial velocity, smaller the mass of the incoming steel ball, larger the susceptibility, higher the output velocity of the last ball. And to get maximum output velocity, the number of steel balls on the left hand side of the magnetic ball should be zero. Nevertheless, the process of collision is so complex that it is difficult to describe it quantitatively and calculate how many steel balls should be put on the right hand side of the magnetic ball. Hence we need to find the number of steel balls on the right hand side of the magnetic ball by the following experiment.

Apparatus and Methods

As shown in figure 2, the track, which includes a curved part and a level part, is made of organic glass. We can change the initial velocity of the steel ball by releasing it from different height of the track. There are several steel balls (mass: 16.822g, radius: 7.5mm) and one magnetic ball (mass: 16.205g, radius: 7.5mm) on the track, and four photoelectric gates are settled on both sides of the balls to measure the velocities. Digital Gauss meter [1] , whose error is 0.05 T, is used to measure the magnetic flux density.

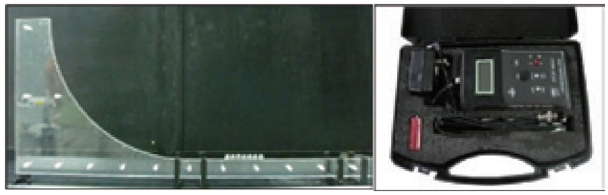

Figure 2. The apparatus used in the experiment and the Gauss meter.

The magnetic flux density of a lone magnetic ball and with different numbers of steel balls was measured to prove the effect of steel balls on the magnetic field of the magnetic ball. We also changed the total number of steel balls, the location of the magnetic ball, the initial velocity of the incoming ball and measured the output velocities.

Experiment

The magnetic flux density on the axis direction of the magnetic ball was measured by digital gauss meter. As shown in figure 3, the magnetic flux density B of a lone magnetic ball decreased quickly. Nevertheless, it increased significantly when more steel balls were put on the right hand side of the magnetic ball.

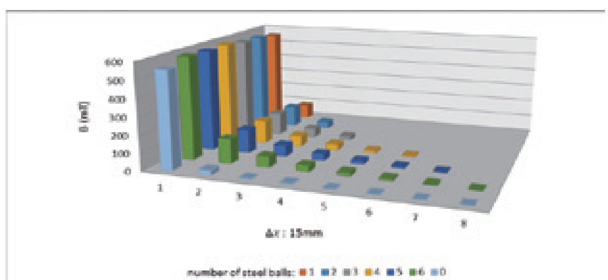

Figure 3. Magnetic flux density as a function of distance and the number of steel balls.

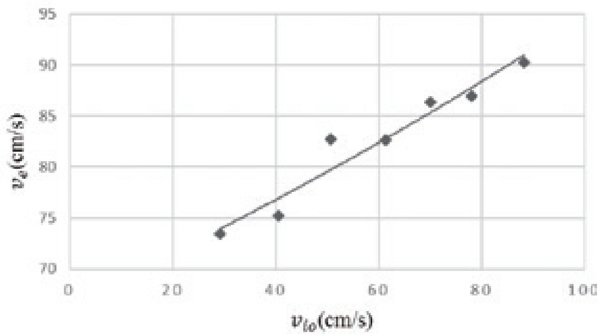

Figure 4. The output velocity v_e as a function of the initial velocity v_{i0} . when n_i = 0, n_e = 2. n_i .

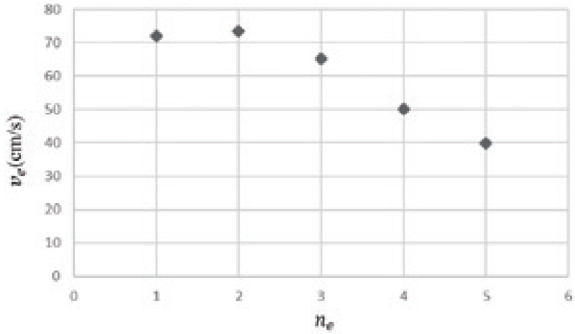

Figure 6. The output velocity v_e as a function of n_e. when n_e=0, v_{i0} = 30.00 ± 0.50 cm/s.

To investigate the key reasons that affect the output velocity, at first, we changed the initial velocity the incoming ball when there were two steel balls and no steel ball on the left and the right side of the magnet. As shown in Figure 4, the output velocity v_e rises with the initial velocity v_{i0} of the incoming ball, and with v_{i0} increasing, the difference between v_{i0} and v_e gets smaller and smaller. The result is consistent with our theoretical result.

After that we changed the number of the steel balls on the left side of the magnet n_i from 0 to 3. As you can see in Figure 5, when n_i=0 and v_{i0}=30cm/s, v_e is over twice v_{i0}, and v_e decreases significantly with n_i. Therefore, to get the maximum v_e, there should be no steel ball on the left side.

At last, to find the best number on the right side of

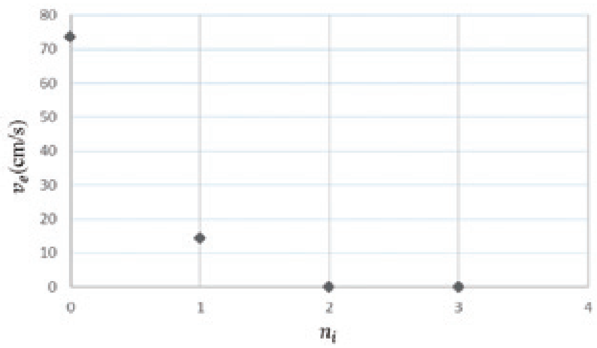

Figure 5. The output velocity v_e as a function of n_i. when n_e=2, v_{i0} = 30.00 ± 0.50 cm/s.

the magnet n_e from 0 to 5, and the result is shown in Figure 6. Apparently, n_e increases when n_e increases from 1 to 2, but then decreases when the number continues increasing. Hence v_e gets its maximum value when n_e=2.

Conclusion

In our work, the magnetic scalar potential is the first time used to research the Magnetic cannon. It makes more clearly that the parameters effect on the maximum speed of the system. Form the result of our theoretical analysis, we find that the mass, initial velocity of the incoming ball, the susceptibility and location of the magnetic ball can affect the output velocity of the last ball.

The output velocity increases with the susceptibility of the magnetic ball and the initial velocity of the incoming ball and secures its maximum value when there is no other steel ball on the left hand side of the magnetic ball. We also find there is the optimal number of steel balls on the right hand of the magnetic ball by our experiments, which is consistent with the reported results [2]. In our experiment, it is 2 for our system.

References

[1]. http://www.cnrinch.com/gauss-meter.htm

[2]. Chittasirinuwat O, Kruatong T, Paosawatyanyong B. More fun and curiosity with magnetic guns in the classroom. Physics Education. 2011, 46(3):318.

Acknowledgements

The author acknowledges Lv Zhaozheng for offering us important idea. I would also like to thank teacher Wang Jin and teacher Chen Ping for providing photoelectric gates and Digital Gauss meter;

This work was supported by the National Science Fund for Talent Training in Basic Sciences under Grant No.J1103208.