To find the physical sense of conditions (14)–(15), the definition of constant \( \gamma \) should be recalled. First of all, both conditions require that

\[ \gamma \ll 1 \]

which means that the initial length of string is smaller, that the distance between closest pendulums. Then, condition (14) means, that the difference between phases of 2 neighboring pendulums is small enough:

\[ \Delta \varphi \ll \frac{d}{R} \]

This is the case of linearized equation of motion. And finally, the condition \( \gamma \ll \ll 1 \) requires to use string with almost zero initial length:

\[ l_0 \ll d \]

To conclude this part, it is often thought that widespread models fit everywhere without any remarks instead of checking it out. This example shows, that every time developing theoretical model, the author should be very careful: when taking model invented by previous researches, he carefully revise all the assumptions made during the derivation of the model, because they can sometimes be inappropriate for the case. And if he takes his own model, every step should be accurately checked.

Conclusions

This task illustrates very interesting and rather complicated phenomenon, which can be met in every part of physics. It shows that our world is not so simple to describe, not all phenomena can be theoretically predicted using already created models.

As for our particular study, we have managed to create a setup, which can demonstrate such interesting phenomena. This setup has 2 absolutely different types of behavior, and both of them were studied and described. And since there are well-known parameters to characterize propagation of waves in any medium, we have chosen propagation velocity as a characteristic, which better describes this process, and investigated influence of different parameters on this value. And finally, we have managed to create 2 different models, which describe this system from mechanics point of view, as well as wave theory. Also we have considered the limitations of a well-known theory applicable for this case, and discussed the physical background for these limitations.

Finally, we have obtained the following results. This system can represent 2 possible types of waves: dispersive waves and solitary waves. Properties of these motions are absolutely different: dispersive waves are defined by so cold dispersion relation and solitary waves are stable and not sensitive to small fluctuations – that is also defined by medium properties.

References


2013 Problem 13: Honey Coils
Thin liquid rope coiling

Abstract

A thin, downward flow of viscous liquid, such as honey, often turns itself into circular coils. This phenomenon was explored and a stream of honey falling vertically from various heights tends to coil with specific behaviours that are predominantly influenced by viscous (\( F_V \)), gravitational (\( F_G \)) and inertial (\( F_I \)) forces. There is an existence of three distinct regimes of liquid rope coiling, the viscous (V), gravitational (G) and inertial (I) regimes. Coiling in these regimes are stable and can be characterized in several ways. Recent studies have shown a fourth regime, known as the inertia–gravitational (IG) regime which is a transitional regime between the gravitational and inertial regime. Coiling frequencies in this regime are multivalued and unique coiling behaviours are observed, such as figure of eight coiling which changes the sense of rotation.

Scaling laws was used to determine the coiling frequency in the different regimes, as well as estimate the forces present within the regimes. The purpose of this study was to experimentally verify the existence of the four regimes as well as coiling within those regimes indicated in the scaling laws. The multivaluedness within the inertia–gravitational regime was also investigated.

Keywords

circular coils, regimes, multivalued, scaling laws, viscous, gravitational, inertial

Introduction

The purpose of this report was to study the motion of a thin, downward stream of honey which turns itself into circular coils. In this study, honey with kinematic viscosity (\( \nu \)), density \( \rho \) free flows out from an orifice of radius \( a_0 \) from various heights (H) at an approximately steady volumetric flow rate (Q) onto a smooth flat plate. The radius of the jet directly above the coil portion of the system is defined as \( a_1 \), and that the radius of the honey coil is defined as \( R \). Previous studies developed scaling laws that describe...
the frequency of the coils in regards to different coiling regimes.

**Experimental Setup**

In this study, honey with dynamic viscosity $\nu$ and density $\rho$ flowed through an aperture of radius $a$ from different heights, $H$, at an approximately steady volumetric flow rate, $Q$, onto a flat surface. The apparatus shown in Figure 2 was consistent in measuring the variables associated with this experiment. The top of the apparatus was holder that held various volumes of honey, but each volume was consistent for each set of trials. Thin acrylic plates are put onto the platforms where the honey fell on. The masses of the plates were recorded, then were first placed near the hole, and subsequently the fall heights was increased. From then on, honey was allowed to flow through $a_0$ and its motion was recorded using a high speed camera of 210 frames/second. Afterwards, the masses of the honey and the plate were measured. The coating frequency, coating radius, $a_1$, and the coil radius, $R$, were measured using the software Tracker[13]. A scale reference was used during the video, which allowed for precise raw data. Calculations of the regimes and other aspects were accomplished using Excel. In order to make the experiment safe, the apparatus was made from plastic and the camera was always held on a tripod. Refer to Figure 1 for the experimental setup.

![Fig 1: Experimental apparatus and setup.](image1)

![Fig 2: Terminology for coiling system that was used for the remainder of the report.](image2)

**Theory**

Honey is a highly viscous fluid coils as it falls onto a surface. It is a thixotropic non-Newtonian fluids, thus the viscosity of honey decreases with the length of time a shear force is applied (Fluids, 2010)[4]. Also, honey is a viscoelastic material that exhibits both viscous and elastic characteristics when undergoing deformations. Honey resists shear flow and strains linearly with time, and the elasticity of honey means that it will return to its original form (Thixotropic, 2011)[12]. Therefore, these viscoelastic materials exhibit a time dependent strain (Extensional viscosity and elasticity of fluids, 2001)[3]. However, for the purposes of this report, the viscosity of honey was assumed constant throughout each experiment, and the time the honey was subjected to flow was minimized.

Honey flows slowly, thus it has a laminar flow. It resists a transition to turbulent flow and flows in smooth layers (Transition and Turbulence, n.d.)[14]. In order to determine whether honey has laminar flow, the dimensionless Reynolds number was used to calculate the type of flow of honey.

$$Re = \frac{2Q}{\pi a_1 \nu}$$

The critical Reynolds number for viscous coil was 1.2 (Varagnat, Majmujar, Hartt & McKinley, 2010)[15].

When a viscous fluid is poured onto a surface, it will begin to coil or fold upon itself, as these liquids take a path of least resistance. This is a phenomenon known as ‘cylindrical viscous jet buckling’ (Batty & Bridson, 2008)[1]. According to Bridson & Batty (2008), the falling fluid above the coil portion of the system and the viscous pile below apply opposing forces, but the surrounding air applies little or no resistance, thus causing the fluid to bend or bow out to one side. Essentially, if the jet is moving faster than the rate at which the viscous pile below can absorb it, the jet will slow down, and widen a short distance above the pile. Thus, each particle of fluid that passes through the narrowest section must slow down, and the opposing force applied by the viscous pile and jet will cause greatest stress in the narrowest section as there is the least surface area, resulting in jet buckling. Therefore, jet buckling occurs initially, then more fluid enters the buckled region, and finally jet moves in a circular motion around the central axis of the stream.

Once the liquid comes into contact with the flat surface, it starts to form a helical coil and angular velocity, $\Omega$, occurs, as seen in Figure 2 (Molina & Hertzberg, 2011)[7]. The liquid forms a liquid rope coil that comprises of a long, nearly vertical “tail”, which is the portion of the flow that precedes the bend that produces the coils, and a helical coil of radius $R$ on the flat surface (Fry, McGuire & Shah, 2008)[5]. Motion of the rope is based on buoyancy, the imposed volumetric flow rate which is resisted by the viscous force and inertia. Surface tension is also included, but typically has only a small (few percent) effect on the coating frequency (Ribe, Habib & Bonn, 2007)[9]. Hence, for this report, surface tension was neglected.

The coating frequency was determined by the viscous, $F_V$, gravitational, $F_G$ and inertial, $F_I$, forces in the coil portion of the viscous rope (Fry, McGuire & Shah, 2008)[5]. At low fall heights, the jet spreads homogeneously onto the plate forming a steady stagnation point, which is driven by a balance between the imposed volumetric flow rate, gravitational acceleration, and the viscous resistance to deformation of the fluid. As the fall height increased, the viscous jet leaves the stagnation point and starts buckling under the compressive stress in which coiling begins (Varagnat, Majmujar, Hartt & McKinley, 2010)[15].

Under the assumption that the coating radius of the jet is constant throughout the system, the frequency, coating radius, and the radius of the coil rope are connected by the conservation of liquid volume

$$Q = \pi a_1 R^2$$

(Varagnat, Majmujar, Hartt & McKinley, 2010)[15]

The forces acting on a viscous jet such as honey can be characterized into three main regimes – viscous, gravitational and inertial regimes, with a unique fourth regime during transitional state between gravitational and inertial regimes known as the inertia-gravitational regime (Batty & Bridson, 2008)[1].

Coiling in the viscous regime occurred when gravitational and inertial forces weree negligible. The viscous forces are somewhat balanced by gravitational forces in the gravitational regime, but inertial forces are negligible by comparison. In the inertial regime, the coating inertia of the honey are approximately balanced by the resistive viscous forces and thus, gravitational forces are neglected. The inertia-gravitational regime experiences coiling when gravity and the inertial force of the coil portion of the honey somewhat balance the viscous force (Fry, McGuire & Shah, 2008)[5].

The coating frequencies and regimes are dependent on the stream radius, the flow rate, the fall height, and fluid’s kinematic viscosity and density. The dependence
of these variables essentially represents a balance between buoyancy and viscous bending resistance forces. The balance or unbalance of these forces acting on the coil portion of the jet causes the coiling frequency first to decrease, and subsequently increase with increasing height. Coiling is governed by scaling laws involving a balance between rotational inertia and the viscous forces resisting the bending of the rope.

Steady coiling occurs and the viscous, gravitational and inertial forces acting of the jet of honey are balanced (Brun, Ribe&Audoly, 2012, p38)[2]. These regimes appear clearly on plots of the dimensionless (scaled) coiling frequency, \( \Omega_C \), versus the dimensionless (scaled) fall height, \( \tilde{H} \).

According to the paper by Brun, Ribé&Audoly (2012), the two dimensionless parameters can be mathematically represented as:

\[
\hat{H} = H \left( \frac{g}{v^2} \right)^{1/3} \text{ and } \Omega_c = \Omega v \left( \frac{g}{v^2} \right)^{1/3}
\]

and in the overall system, honey first experiences coiling in the \( \hat{H} \approx 0 \) regime. For small heights, \( \hat{H} < 0.08 \), coiling occurs in the viscous regime. At low fall height, the tail of the jet is experiencing buckling, and the viscous torque caused by the fixed vertical orientation of the aperture controls the motion of the jet. The torque balance leads to the scaling laws for the coiling radius, \( R_C \), and magnitude of the viscous force per unit rope length, \( F_C \).

\[
F_C \equiv F_0 \approx \rho a_0^4 u_1 \Omega_y \approx \rho a_1^4 U_1 \Omega_y \approx \rho \frac{v^4}{u_1} U_1 R^4 \approx \frac{F_y}{\rho}\]

From this, it follows: \( \rho a_1^4 U_1 R^4 \approx \frac{F_y}{\rho} \). This gives rise to the inertial coiling frequency, which is proportional to:

\[
\Omega_{c} = \left( \frac{g \Omega_y}{u_1^2 v^4} \right)^{1/4}
\]

Thus, \( a_1 \) and \( a_2 \) can be determined using the relationships below.

\[
F_y \approx 0.4 \rho \alpha \sin \theta y \approx \rho a_2 \Omega y^3 \psi
\]

\[
F_y = \frac{F_C}{\rho a_1^4 U_1 R^4} \approx \frac{F_y}{\rho a_1^4 U_1 R^4}
\]

\[
\rho \approx \frac{F_y}{\rho a_1^4 U_1 R^4}
\]

\[
\Omega_y = \left( \frac{g \Omega_y}{u_1^2 v^4} \right)^{1/4}
\]

The coiling frequency in this regime is approximately:

\[
\frac{F_y}{\rho a_1^4 U_1 R^4} \approx \frac{F_y}{\rho a_1^4 U_1 R^4} \approx \frac{F_y}{\rho a_1^4 U_1 R^4}
\]

Thus, the scaling laws show

\[
F_y \approx \rho a_1^4 U_1 R^4 \approx \rho a_1^4 U_1 R^4 \approx \rho a_1^4 U_1 R^4
\]

\[
\Omega_y = \left( \frac{g \Omega_y}{u_1^2 v^4} \right)^{1/4}
\]

Thus, it requires:

\[
\rho \approx \rho a_1^4 U_1 R^4
\]

Thus, the scaling laws show

\[
F_y \approx \rho a_1^4 U_1 R^4 \approx \rho a_1^4 U_1 R^4 \approx \rho a_1^4 U_1 R^4
\]

\[
\Omega_y = \left( \frac{g \Omega_y}{u_1^2 v^4} \right)^{1/4}
\]

\[
\rho \approx \rho a_1^4 U_1 R^4
\]

\[
\Omega_y = \left( \frac{g \Omega_y}{u_1^2 v^4} \right)^{1/4}
\]

Thus, the scaling laws show

\[
F_y \approx \rho a_1^4 U_1 R^4 \approx \rho a_1^4 U_1 R^4 \approx \rho a_1^4 U_1 R^4
\]

\[
\Omega_y = \left( \frac{g \Omega_y}{u_1^2 v^4} \right)^{1/4}
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Thus, the scaling laws show

\[
F_y \approx \rho a_1^4 U_1 R^4 \approx \rho a_1^4 U_1 R^4 \approx \rho a_1^4 U_1 R^4
\]
the viscosity of the liquid. The equations basically implied that as long as the fall height increased, the coiling frequency would also increase. However, deeper understanding showed that if the aspect ratio of $a_1/a_0$ was always approximately 1, than coiling would not increase as the viscous stream would still be at the stagnation point. Thus, these graphs have limitations and certain conditions must be reached in order to effectively draw conclusions.

Figures 3 to 6 are logarithmic graphs and show that the ratio of $a_1$ to $a_0$ was almost equal to unity for Tests No.1 and/or 2 of all four sets of experiment; then it decreased to less than 0.1. In addition, a band of data points were seen when the scaled height was large. In general, the hypothesis was supported as when the scaled dimensionless coiling frequency increased, the scaled falling height also increased under the condition that $a_1/a_0$ would decrease.

The coiling frequency trend in the Figure 3 was only relatively consistent when compared to the consistent coiling frequency trend in Figures 4, 5, and 6. In Discussion 2, it was seen that the aspect ratio of $a_1/a_0$ was also small, which meant that the honey stream was not consistent in when falling from greater fall height. Multivalued coiling, according to this representation of the coiling, frequencies versus height was only seen in Figure 8. It occurred at $H\approx 1.77$, which was within the domain of the inertio-gravitational regime. A strength of this analysis was that the dimensionless coiling frequency versus dimensionless fall height was scaled successfully as the aspect ratio of $a_1/a_0$ decreased when the dimensionless fall height decreased.

To identify the coiling in the regimes was to use the $\Omega/I\Omega_g$ ratio which was proposed by Ribeet et al. (2007) and confirmed by Fry et al. (2008) versus the scaled dimensionless height that was proposed by Ribe (2004). Figures 7 to 10 show relationship between the $\Omega/I\Omega_g$ ratio and the falling height for all four experiments, and the Tracker screenshots for representative tests.

Take Experiment 1 as an example, the regime classifications in Figure 7 were exactly coincident with the observations in experiment 1 and showed that Tests No.1 to 3 were in the viscous and gravitational regimes and the Tests No.4, 5 and 8 in the inertio-gravitational regime.

Regarding the Test No.11 of the Experiment 2 (H = 10.31cm), both the scaled dimensionless height and the $\Omega/I\Omega_g$ ratio methods failed to demonstrate that the coiling was actually in the inertio-gravitational regime, instead of the inertial regime, see Figure 8. The graph produced an interesting observation, which was that multivalued coiling occurred in the inertial regime at a height of 10.33cm. As seen in Figure 16, the $\Omega/I\Omega_g$ ratio was 3.486, which was not significantly different from the maximum ratio for the coiling to be in the inertio – gravitational regime. Thus, the honey was in a unique situation where the inertial forces were not approximately balanced by viscous forces, but by some gravitational induced stretching as well, thus causing multivalued coiling.

In Figure 7, the coiling frequency at a height of 9.03cm (also see Test No 8) was seen in the inertio – gravitational regime, even though two previous coiling frequencies had entered the inertial regime. This suggests that the flow rate at that particular instant was not constant, and thus, the approximate balance between the viscous and inertial forces were disrupted and some gravitational force entered the system in order to balance the dominant viscous forces.

In Figure 9, coiling in the IG regime was approximately constant, that is, 1.96, 1.92 and 1.93 for fall height of 5.83cm, 6.41cm and 7.01cm respectively (an example of IG coiling is shown in the image Test No 7 in Figure 17). In regards to the height of 6.41cm, the image of Test No 7 showed that the jet changed its sense of rotation as it coiled in a “figure of eight” manner. Thus, the coiling frequencies could have been approximately constant as different coiling rotations and changes meant that there were different distances travelled by the jet, and that would have led to the similar coiling frequencies.

As seen on the last image in all four figures, there were no number of coils in the viscous pile before the honey collapsed. This was when the honey entered the inertial regime, which means that the ratio
of $a_0/a_0$ was small. A small $a_0/a_0$ aspect ratio suggests that there was more viscous bending as the jet was extremely thin, thus the pile of viscous fluid absorbed less volume of honey because the coils were small, and thus, As a result, more coils could be piled on the viscous pile.

In all the above graphs, coiling occurred predominantly in the inertial regime, and only one or two occurred in the viscous and gravitational regimes. Perhaps, the coiling entered the inertial regime because $a_0$ was reasonably small in the experiments, and thus, Hence, this significant stretching occurred in the viscous jet, which caused the coiling to enter the inertial regime at a faster rate.

These graphs were a more precise and accurate representation of when coiling occurs in the regimes occur as coiling in all the regimes are shown. A limitation in these graphs was that the coiling frequency at which viscosity or gravitational forces could not be seen directly, and had to be determined from previous analyses. As seen from the above figures, ratio of $\Omega_1/\Omega_g$ versus the fall height could only determine where the combined gravitational and viscous regimes would be and not where the individual G and V regimes would be.

This method is to use the regime forces to identify a coiling is in the viscous force, gravitational force and the inertial force. From previous theory, it is known now have learnt it is known that

a) Viscous regime: the viscous force is dominant, while the gravitational and inertial forces are negligible.
b) Gravitational regime: viscous force is approximately balanced by the gravitational force, but the inertial force is negligible.
c) Inertial regime: viscous force is somewhat balanced by the inertial force, while the gravitational force is negligible.
d) Inertia-gravitational regime: the viscous force is balanced by both gravitational and inertial forces, that is, $\frac{\Omega_1}{\Omega_g}$, i.e., at this stage the motion of coils is governed by a balance of gravity, centripetal force of the coil, and the axial tension within the stretching. It means that gravitational and inertial forces are within the same scale at the inertia-gravitational regime.

The hypothesis stated that a graph of the forces versus the $\Omega_1/\Omega_g$ ratio should show the forces within the different regimes. The viscous regime should have predominantly viscous forces, and the gravitational forces should decrease as the jet progresses through the regimes, whereas the inertial force should subsequently increase. The hypothesis was somewhat supported somewhat to some extent, and the findings were as analysed below.

The different forces were graphed in relation to $\Omega_1/\Omega_g$ (independent variable) as the latter provided a more accurate representation of coiling within the four distinct regimes. The horizontal pink dashed lines show the transition between the regimes. As mentioned in the theory, $\Omega_1/\Omega_g$ was between 0.7 and 2.0. Figures 11 to 14 give the relationships of the 3 forces with the $\Omega_1/\Omega_g$ ratio for all four experiments. From these four figures, we are able to summarize the following common views, were summarized.

For the tests marked as viscous regime, the viscous forces are very high so that they have exceeded the viscous force scale (the secondary axis in Figures 11 to 14). The other values were not included, as showing the extra points would disrupt the ‘guide of the eye’. As seen in Figures 11 - 14, the viscous force was the most dominant force in honey coiling throughout the regimes. This supported the theory and the viscous forces should be dominant as it is the viscosity of the fluid that controls the motion of honey. The scattering of the viscous force below all the figures below suggest that the flow rate of honey was not constant, and that would change the magnitude of the viscous force in the jet.

The above four figures essentially showed that, within the gravitational regime, (G) the gravitational force was much higher than the inertial force, which was negligible as stated in the theory. The viscous forces in the gravitational regime in all the figures are greater than the gravitational forces, which were in accordance with the theory as viscous forces should always be the dominant one in the system. It was also noticed in Figures 11, 12, 13, and 14, that the gravitational force would decrease as the stream of honey transitioned into the other regimes, which was in agreement with the theory.

In the identified inertia-gravitational coiling (IG), the magnitudes of the gravitational and inertial forces were very close, while at the boundary of IG and I regimes, the inertial forces were higher than the gravitational forces, refer to Figure 11, 12, 13 and 14. This phenomena does make sense, as the transition from IG to I regime was a smooth and gradual process, rather than a sharp jump, and the inertial forces become more significant than the gravitational forces.

It is clearly evident in the inertial regime (I), that the inertial forces are very close to the viscous forces, while the gravitational forces were considered negligible. In Figure 11, it can be seen that the inertial forces gradually increased from the viscous regime to an $\Omega_1/\Omega_g$ ratio of 4.753, which meant that the coiling of honey was in the inertial regime. After a $\Omega_1/\Omega_g$ ratio of 4.753, the inertial force was scattered, which suggests that the flow rate was not constant. Figure 12
followed a very similar trend, but the inertial force was greater than the viscous force in some parts. Proposes that the jet was stretched significantly. Figures 13 and 14 showed that as honey coiling progressed through the regimes, the inertial forces increased, which was in accordance with the theory.

Similar to the $\Omega_f/\Omega_g$ ratio method, this technique of graphing the forces within the regimes was an appropriate and effective way to identify which forces were dominant in the regimes. Also, the theory and hypothesis was easily compared to the graphs. A limitation in this graph was that if the viscous forces were too high, then the values had to be excluded; otherwise, the other forces would not be clearly seen.

### Conclusion

Analysis one showed the relationship; the dimensionless coiling frequency would increase. This supported the hypothesis and. This relationship was $a_j/a_k$ decreased as the other factors increased was satisfied.

Analysis two included graphs of $\Omega_f/\Omega_g$ versus the fall height. It shows when the viscous would enter different regimes. Multivalued coiling was observed in some parts. However, gravitational forces subsequently decreased throughout the regimes and the inertial forces increase.

Suggestions for future improvements are proposed here in hopes of enhancing the design of the apparatus as well as the depth of analysis regarding honey coiling, which involved factoring in the time–dependence as well as viscous properties. More efficient measuring equipment is also needed to ensure precise measurements.

Honey coiling revolved around four regimes, the viscous (V), gravitational (G), inertio – gravitational (IG) and the inertial (I) regimes. Multivalued coiling is observed in the IG regime as there is a change in sense of rotation.

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### REFERENCES


Abstract

Many fluid flows in nature are governed by buoyancy-driven convection. One such classical convection theory, Rayleigh-Bénard Convection (RBC), has demonstrated great success in predicting storms and atmospheric current flows. However, a variety of other natural phenomena, such as geomagnetism, convection in the Arctic Oceans, and behaviors in the interiors of gaseous celestial bodies, are instead governed by its rotating counterpart, rotating Rayleigh-Bénard Convection (rRBC), which is a non-linear phenomenon where the traditional RBC is observed on a rotating frame of reference. rRBC is a non-trivial problem worthy of investigation due to its resemblance to many non-linear systems, spanning climatology, oceanography and astrophysics. Stable states of a rRBC fluid and their evolution have been treated much in depth by current literature, both experimentally and theoretically. However, a linear stability analysis, instead of the traditional variational approach proves effective in locating the critical Rayleigh number beyond which convection sets in. This paper establishes a mathematical model describing this boundary of stable state using a multi-scale perturbative (MSP) approach.

Keywords

Rayleigh-Bénard Convection, rotating frame of reference, multi-scale perturbation.

Introduction

Convection is a mechanism whereby hot fluid rises and cold fluid sinks which gives rises to efficient heat transport between the upper surface of the liquid and the lower one. Rayleigh-Bénard convection (RBC) characterizes the buoyancy-driven nature of this phenomenon and surface-tension-driven convection is thus not considered. The control parameter Rayleigh number \( R \) as shown in equation (1), indicates the magnitude of thermal driving force of the system,

\[
R = \frac{g \alpha T_0 d^3}{v \kappa}
\]