 Correction Technique for van der Pauw Method

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Abstract

The van der Pauw method makes many assumptions, one of which is the sample being tested having not holes. By constructing a virtual resistor network to represent the sample and simulating the van der Pauw method, it is demonstrated how a numerical correction factor can be found. This numerical correction factor is applied to the conductivity of the non-ideal sample as measured by the van der Pauw method to find the conductivity of the material being tested. This process is then experimentally verified using different shapes and connection orientations. It is also noted that for some shapes with holes the van der Pauw method gives a result within experimental error of the conductivity of the material tested.

1 Problem Statement

It is known that conductivity of a material can be measured independently of the sample shape, as long as the sample has one border (no holes). To what extent can such a method be applied? Investigate and explain such measurements if the sample has holes.

2 Introduction

To use the van der Pauw method, the following equation (Eq. (1)) is solved and has been demonstrated to be true [1][2]. Note that conductivity (\( \sigma \)) is used rather than the more commonly used inverse electrical property, resistivity (\( \rho \)).

\[
\exp(\pi \sigma R_{AB,CD}) + \exp(\pi \sigma R_{BC,DA}) = 1
\]  

(1)

Where \( t \) is the material thickness (m) and \( \sigma \) is the conductivity of the material (S/m). \( R_{AB,CD} \) is calculated as \( R_{AB,CD} = \frac{V_{CD}}{I_{AB}} \) where \( I_{AB} \) is the current contact points A and B, and \( V_{CD} \) is the potential difference measured across contact points C and D. \( R_{BC,DA} \) is calculated by passing current through
3 Simulation Technique

Finite element analysis is used to simulate the sample by constructing a virtual resistor network. For simulation of a square sample, the network has dimensions of 400 cells along each side which gives a conductivity with 0.001% of the actual conductivity. A cell consists of four identical resistors, as shown in FIG. 2. The conductivity of the resistor network is defined as the inverse of the resistance of the network resistors. The simulated values for $R_{AB,CD}$ and $R_{BC,DA}$ are found using an electrical simulation program. To test a sample of a given shape for the van der Pauw method, a digitized image allows for the shape to be simulated with a virtual resistor network. The equation for the van der Pauw method is solved for the non-ideal network being simulated.

The ratio between the actual conductivity of the network and the conductivity found by solving Eq. (1) is defined as the correction factor, $k$ (Eq. (2)).

$$k = \frac{\sigma_{\text{network}}}{\sigma_{\text{simulated}}}$$

(2)

For an ideal sample with no holes, $k = 1$. For a sample with holes, $k \geq 1$, as the conductivity calculated using a sample with holes will be less than the conductivity of the material.

4 Experimental Set Up

Testing was completed on copper printed circuit boards with a thickness of $32 \pm 2$ mm. By completing testing on copper, the expected value for the conductivity of the sample being tested should be within experimental error of 59.5 MS/m[^3]. The tested shapes were drawn in a CAD program and then transferred onto copper boards using Press and Peel film. The excess copper was etched away using Ferric Chloride. Each shape had connection tabs added to edge of the sample to allow for wires to be soldered. The wires were then connected in the circuit shown in FIG. 3. A photograph of the experimental set up is shown in FIG. 4. Note that with a hole in the sample the conductivity calculated by the van der Pauw method will tend to be lower than the actual conductivity of the material.
5 Experimental Results

A wide range of tests were carried out in which current, sample shape and size, hole shape, size, number, position and connection locations were all varied independently. For the conductivity of the sample as measured by the van der Pauw method, the horizontal dashed line shows the known conductivity of copper.

Using the sample shown in FIG. 5, different connection orientations can be tested (shown in TABLE I), for a central hole on a square sample. The conductivity found by the van der Pauw method for the orientations is shown in FIG. 6.

A sample with a non-central hole (FIG. 7) is tested with the connection orientations in Table tested, with the conductivity measured using the van der Pauw method shown in FIG. 8. The samples used are shown in FIG. 9.

Three arbitrary shapes were tested, all with the same connections tested (TABLE II).

The three shapes had different holes cut out of them. They were defined as, arbitrary shape with two holes (FIG. 10a), arbitrary shape with one large hole (FIG. 10b) and arbitrary shape with one hole (FIG. 10c). The conductivity as calculated using the van der Pauw method for the arbitrary shape with two holes is shown in FIG. 11 and for the arbitrary shape with one large hole is shown in FIG. 12. The conductivity as calculated by the van der Pauw method for the arbitrary shape with one hole (FIG. 10c) shown in FIG. 13 and discussed in Example Correction method in detail.

Note that for each of the connection placement and shapes (FIG. [6, 8, 11, 12]) some of the conductivities as measured by the van der Pauw method are within experimental error of the conductivity of the material tested. This means that despite a sample having hole, it has been demonstrated that the van der Pauw method can still be applied in some circumstances. However this would not be known until the correction factor had been calculated using the simulation process.

6 Example Correction Method

Using the example of an arbitrary shape with one hole (FIG. 10c, FIG. 13), the conductivity using the
van der Pauw equation is calculated using the measurements of $R_{AB,CD}$ and $R_{BC,DA}$. There are multiple connections made to the board to allow for different connection orientations to be tested (TABLE II). The conductivity as calculated by the van der Pauw method for the different connection orientations of this shape are shown in FIG. 14. The large uncertainty is due to the inherent error in the van der Pauw method, which arises from the very small potential differences across a copper sample, and the measurement of the thickness of the material which is compounded by these measurements being within an exponential function. Note that despite the presence of a hole connection orientations C, F and G give conductivities that are within experimental error of the conductivity of copper.

The shape (FIG. 13) is used as the basis for constructing the resistor network to be simulated. This provides the correction factors as shown in FIG. 15. Note that the connection orientations C, F and G have simulated correction factors close to the ideal value of 1 (TABLE III). This suggests that there are possible connection placements around the edge of a shape that has a hole in it, which could measure the conductivity of the material without applying a correction factor.

By multiplying the conductivity as measured by the van der Pauw method by the correction factor found by simulation, the true conductivity of the board tested is found to be within experimental error of the conductivity of copper, as demonstrated by Eq. (3).

$$\sigma_{\text{corrected}} = k\sigma_{\text{VDPM}}$$

Where $\sigma_{\text{corrected}}$ is the conductivity for the sample after the correction factor, $k$, has been applied.

![Diagram of arbitrary shape with two holes](a)

![Diagram of arbitrary shape with large hole](b)

![Diagram of arbitrary shape with one hole](c)

FIG. 10
to the conductivity that has been measured using the van der Pauw method, \( \sigma_{\text{DPM}} \). This is used on the shape FIG. 10c, resulting in a corrected conductivity graph FIG. 16. Note that each if the corrected conductivities are within experimental error of the conductivity of copper.

7 Comparison

A comparison of the corrected conductivities as calculated using the van der Pauw method on non-ideal samples and using the simulated correction factor (Eq. (3)) is shown in FIG. 17. This demonstrates how for most of the non ideal samples tested, the use of the correction method is applicable as each corrected conductivity is within experimental error of the conductivity of the material tested. The only exceptions to this are two of the rotation tests where the expected value falls just outside the error range. This may be due to slight asymmetry in the placement of connections when the board is rotated compared to the simulation connections. TABLE IV gives brief descriptions of the shapes of each parameter tested. Unless otherwise specified, the hole size and shape described is shown in FIG. 18, a 7 cm square sample with connections in each corner with a 2 cm square hole that is centrally located on the sample.

8 Conclusions

Some of the connection placements for a sample tested that has holes demonstrates that it is possible to calculate a conductivity using the van der Pauw method that is within experimental error of the conductivity the material. In these cases the correction factor as previously defined is close to 1. The use of finite element analysis to construct a resistor network that is representative of the sample being tested can be used to find the conductivity of the sample, even if the sample has holes.
### Parameter Name and Description

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Parameter Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole Size</td>
<td>On a 7 cm square sample with connections in corners, centrally placed square hole with side lengths from 0 cm to 6 cm in 1 cm intervals</td>
</tr>
<tr>
<td>Aspect Ratio of Rectangular Hole</td>
<td>On a 7 cm square sample with connection in the corners, centrally placed rectangular holes of a constant area of 4 cm with side lengths of 0.67, 0.8, 1, 1.33, 2, 3, 4, 5 and 6 cm</td>
</tr>
<tr>
<td>Connection Placement Central Hole</td>
<td>Connections M to U (TABLE I) for shape shown in FIG. 5</td>
</tr>
<tr>
<td>Connection Placement Non Central Hole</td>
<td>Connections M to U (TABLE I) for shape shown in FIG. 7</td>
</tr>
<tr>
<td>Sample Size</td>
<td>Same hole to sample side length ratio as, with side lengths of 3 - 7 cm at 1 cm intervals and maximum side length of 9 cm</td>
</tr>
<tr>
<td>Current</td>
<td>Increasing current on 7 cm square sample with a square 2 cm hole from 0.1 A to 1 A at 0.1 A intervals</td>
</tr>
<tr>
<td>Rotation of Rectangle (over 180°)</td>
<td>Rotation of a central rectangular 0.8 cm by 5 cm hole on a 7 cm square board with connections in corners at 15° intervals over 180°</td>
</tr>
<tr>
<td>Arbitrary Shape with Two Holes</td>
<td>Connections A to H (TABLE II) for shape shown in FIG. 10a</td>
</tr>
<tr>
<td>Arbitrary Shape with Large Hole</td>
<td>Connections A to H (TABLE II) for shape shown in FIG. 10b</td>
</tr>
<tr>
<td>Arbitrary Shape with One Hole</td>
<td>Connections A to H (TABLE II) for shape shown in FIG. 10c</td>
</tr>
</tbody>
</table>

**TABLE IV.** Description of the parameters tested as shown in the legend of FIG. 17.

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**References**


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**Acknowledgements**

Kent Hogan (Onslow College), Michael Pot, Patrick Herd, Felicia Ullstad (MacDiarmid Institute, Victoria University of Wellington), Izzy Bremmer, Zoe Danger Mansell, E. Prof. Joe Trodahl (Victoria University of Wellington) and Dr Ben Ruck (Victoria University of Wellington).
Conclusions


[13] Performing van der Pauw Sheet Resistance Measurements Using the Keithley S530 Parametric Tester Keithley Application Note Series. Number 3180


2016 Problem 12: Van der Pauw Method