

2012 Problem 16: Rising Bubble

Motion and shape of a rising bubble in a tube

Abstract

A bubble rising in a tube reaches the terminal velocity very quickly due to the viscous drag of the surrounding liquid. Because of the no-slip boundary condition of the fluid at the tube wall, the presence of the tube typically increases the drag and therefore, decreases the terminal velocity. The balance between the buoyance force and the drag determines the magnitude of the terminal velocity. Bigger bubble size leads to greater buoyance force, yet the drag becomes bigger, too. The tube effect on the bubble drops as the tube radius increases. Liquid viscosity directly influences the drag and thus it would change the terminal velocity. We observed that the tube material could affect the terminal velocity, too. When the Re is sufficiently large and the bubble size is much smaller than the tube diameter, the bubbles rise in a zigzag fashion and in the high Reynolds number regime, the bubble turns to a non-circular shape.

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Introduction

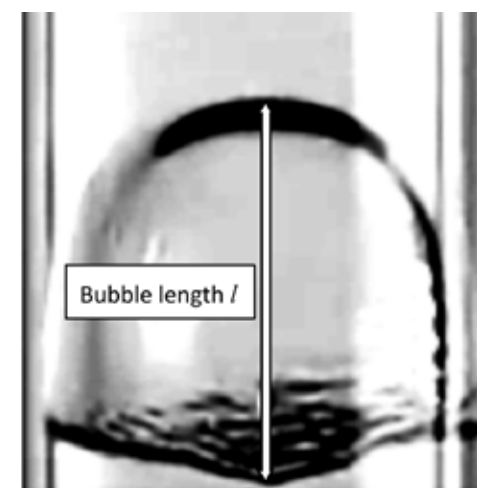
The physics of a rising bubble has been studied for a long time. In the low Reynolds number regime, a rising bubble can be considered as a solid sphere and the viscous drag can be understood using the famous Stokes' formula [Ref.6]. In the high Reynolds number regime, the Stokes' formula is not a good approximation. The drag equation attributed to Lord Rayleigh describes the phenomena more properly [Ref.7]. Moreover, as the bubble rises, its shape deforms and the air-liquid interface permits slip, meaning that this is a difficult free-boundary problem in fluid dynamics. Last but not least, when the bubble rise in a tube filled with liquid, the no-slip boundary condition at the tube wall typically leads to higher drag, which is the focus of this paper.

Experiment setup

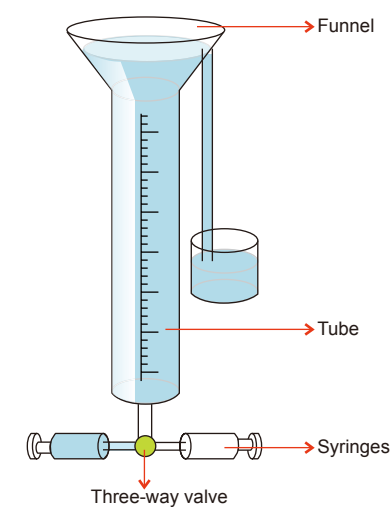
The main purpose of the setup is to produce a single bubble at a time and let it rise through the tube and observe it. The function of the funnel is to control the height of the liquid to fix the pressure. The syringe on the right is used to produce bubbles and the one on the left is for liquid refilling.

Experimental section:

1. Vertical motion of a rising bubble



[Fig.1] A rising bubble in a tube



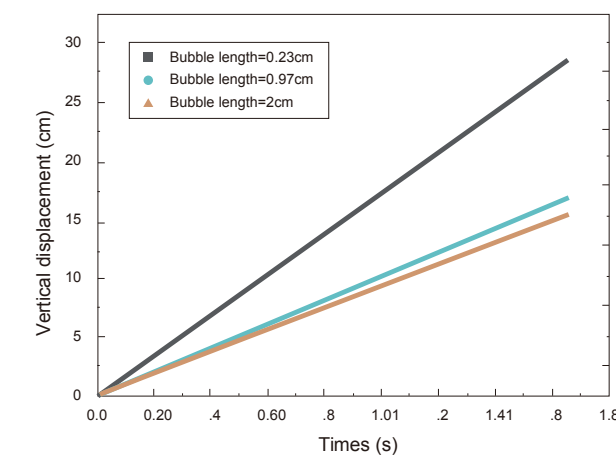
[Fig.2] Experiment setup

I. Terminal velocity (V_T) of a rising bubble

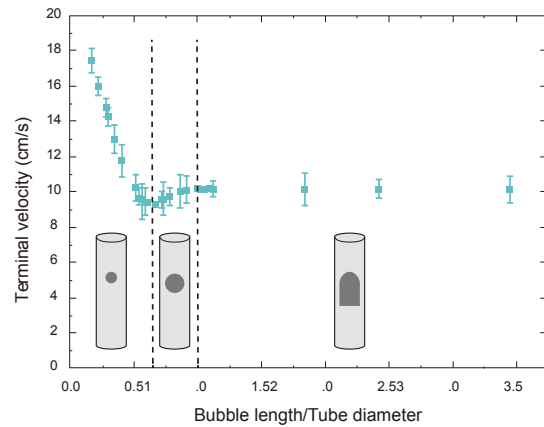
In this part of the experiment we use an acrylic tube having the inner diameter = 1.45cm and only vary the bubble size. We choose the bubble length l to represent bubble size [Fig. 1]. The bubble length refers to the distance from the top to the bottom of the bubble. Figure 2 shows three typical examples of bubble rising in the tube filled with water. The three sizes can represent bubbles whose length is smaller than, approximately to, and bigger than the tube diameter.



[Fig.3] Bubbles of different sizes rising in a tube. The bubble lengths are 0.23cm, 0.97cm and 2.00cm from left to right, respectively. The diameter of the tube is 1.45cm. The liquid in the tube is water and the tube material is acrylic.



[Fig.4] Bubbles rising in a tube with constant vertical velocity. Time=0(s) refers to the moment that the bubble was released. The bubble lengths are 0.23cm (black), 0.97cm (red) and 2.00cm (blue). The tube diameter is 1.45cm. The liquid in the tube is water and the tube material is acrylic. The terminal velocities are 17cm/s, 9cm/s, 10cm/s for black, red and blue points respectively.



[Fig.5] Effect of r on V_T in three regimes. The diameter of tube is 1.45cm. The liquid in the tube is water and the tube material is acrylic. The Reynolds number of the bubbles are between 100 and 1500.

Figure 4 shows the vertical motion of the three bubbles. The vertical displacement is linear in time, meaning that the bubbles reach the terminal velocity V_T almost immediately after release.

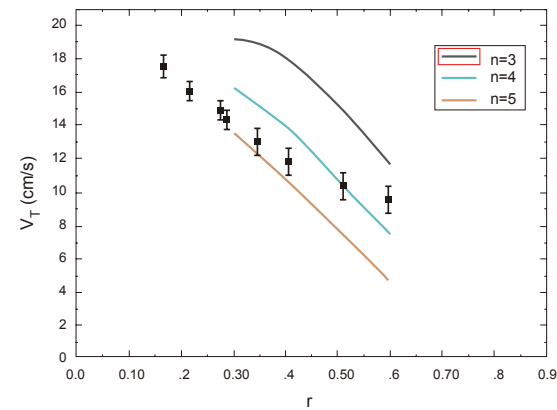
II. The effect of the tube size on V_T

In order to characterize the effect of the tube, we scale the bubble length (l) with the tube diameter (D_t), i.e., let $r = \frac{l}{D_t}$. Figure 5 shows the r -dependence of V_T for a given tube diameter.

The result of this figure suggests that the phenomena of bubble rising in a tube can be divided into three regimes: (a) the small-bubble regime ($0 \leq r \leq 0.6$) where V_T decreases as r increases, (b) the big-bubble regime ($0.6 \leq r \leq 1$) where V_T increases slowly with r , and (c) the Taylor-bubble regime ($r \geq 1$) where V_T is independent of r . The original definition of Taylor bubble can be found in [Ref.4].

A. Small-bubble regime :

For the sake of simplicity, we treat the bubbles in this regime as spheres. As the bubble length increases, the buoyance force also increases. However, the drag rises, too. When the bubble reaches its V_T , The



[Fig.6] The result of [eq.4], $\rho=1000 \text{ kg/m}^3$, $Re=(10^2, 10^3)$, $R_t=0.725\text{cm}$. The graph shows that [eq.4] is a good approximation in the range where $n=3$ to $n=5$.

buoyancy F_B is equal to the drag force F_D , where

$$F_B \approx \frac{4}{3}\pi\rho gR_b^3 \quad [\text{eq 1}]$$

$$F_D = \frac{1}{2}C_D\rho\pi R_b^2V_T^2 \quad [\text{eq 2}]$$

where ρ is the density of liquid, g is the gravitation constant, and C_D is the coefficient of drag.

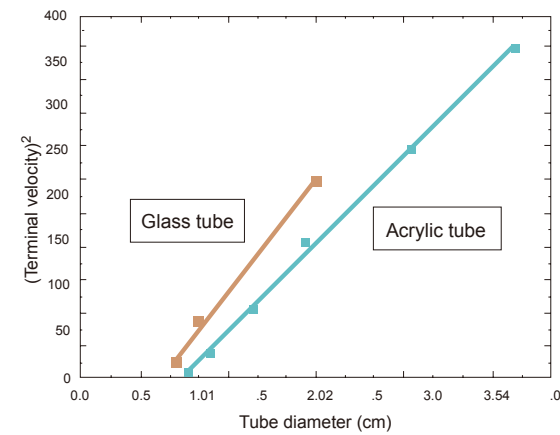
Equation 1 is the result of Archimedes law, which means that the buoyance force equals the liquid weight excluded. Equation 2 is the semi-empirical formula of the drag when the Reynolds number is much bigger than 1.

Reference 3 gives the functional form of C_D for a solid spherical object in a cylindrical tube. For the sake of simplicity, we treat the bubble as a solid sphere and use the following empirical formula generalized from the C_D of Ref.3:

$$C_D = \left(\frac{24}{Re} + \frac{4}{Re^{0.33}}\right)\left(\frac{R_t}{R_t - R_b}\right)^n \text{ for } Re < 1. \quad [\text{eq 3}]$$

The term $\left(\frac{R_t}{R_t - R_b}\right)^n$ shows the fact that while $R_b \rightarrow R_t$, the drag becomes very large.

Combining [eq.1], [eq.2] and [eq.3], we



[Fig.7] Effect of tube size on V_T . The liquid in the tube is water. The tube material is acrylic for the black squares and the formula of the fitting line is: $V_T^2=128(D_t-0.85)$ with V_T in the unit of cm/s and D_t in the unit of cm; The tube material is glass for the red squares and the formula of the fitting line is: $V_T^2=160(D_t-0.65)$ with V_T in the unit of cm/s and D_t in the unit of cm

derive

$$V_T = \sqrt{\frac{8gR_b}{3\left(\frac{24}{Re} + \frac{4}{Re^{0.33}}\right)}\left(\frac{R_t - R_b}{R_t}\right)^n} \quad [\text{eq 4}]$$

B. Big bubble regime

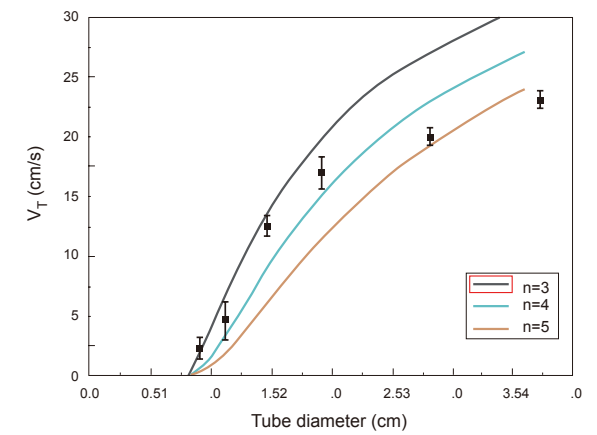
The regime is a combination of small-bubble regime and Taylor-bubble regime. Both sphere and cylinder are not proper approximation for the bubble shape in this regime. Therefore, neither [eq.1] nor [eq.5] (see below) can describe F_B in this regime.

C. Taylor-bubble regime

When the bubble reaches its V_T , The buoyance F_B is equal to the drag force F_D , where $F_B - F_D = 0$. The bubble in this regime is approximately a cylinder. The Archimedes law yields

$$F_B \approx \rho g \pi l R_b^2 \quad [\text{eq 5}]$$

We expect that the shear stress is in proportion to the coefficient of viscosity and depends on the flow velocity, say,



[Fig.8] The effect of tube size on V_T for the bubbles having the same size. The length of the bubble is about 0.80cm. The liquid in the tube is water and the tube material is acrylic. The black squares are the experiment data. The lines are the fitting curves for [eq.4] changing the tube diameter. $\rho=1000 \text{ kg/m}^3$, $Re=(10^2, 10^3)$, $R_b=0.4\text{cm}$. The graph shows that [eq.4] is a good approximation in the range where $n=3$ to $n=5$.

V_T . Because the Reynolds number of the bubble is about 10^3 ; We hypothesize

$$\frac{F_D}{2\pi R_t l} = k\mu V_T^2 \quad [\text{eq 6}]$$

Combining eqs. 5 and 6 and given that $R_b=R_t$ for Taylor bubbles, we derive

$$V_T = \sqrt{\frac{\rho g R_t}{2k\mu}} \quad [\text{eq 7}]$$

Equation 7 shows that in Taylor bubble regime, V_T is independent of length of bubble, and thus it is independent of r .

III. Effect of the tube diameter on V_T

A. Taylor bubbles (In this part of experiment, the bubbles are all Taylor bubbles.)

Ref. 4 suggests that V_T of a Taylor bubble is approximately in proportion to the square root of the tube diameter. However, [Fig.7] shows that there is an offset and that different materials yield different offsets. We think that the wetting property of the tube surface is the main cause.

Same size bubbles (In this part of experiment, the bubbles have the same size.)

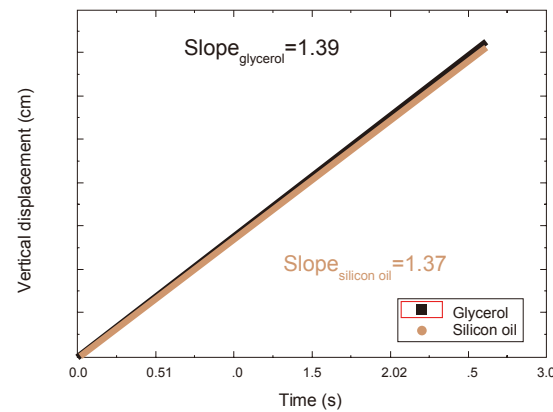
B. Same size bubbles (In this part of experiment, the bubbles have the same size.)

Equation 4. suggests that greater R , leads to larger V_T , as shown in [fig.8.]

IV. Effect of liquid viscosity on V_T

The first part checks the influence of the liquid property on V_T of a rising bubble besides viscosity.

The result shows that viscosity is the main

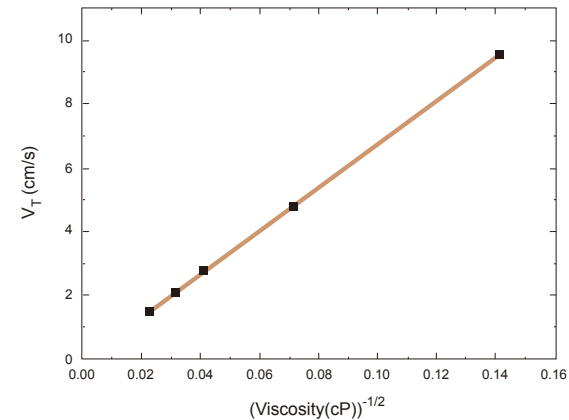


[Fig.9] Influence of liquid property with fixed viscosity on V_T of a rising bubble. The liquids are glycerol ($\mu \approx 945 \text{ cP}$ in 25°C) and silicon oil ($\mu \approx 1000 \text{ cP}$ in 25°C) for the black line and red line, respectively. The bubbles are Taylor bubbles. The diameter of tube is 1.45cm. The tube material is acrylic.

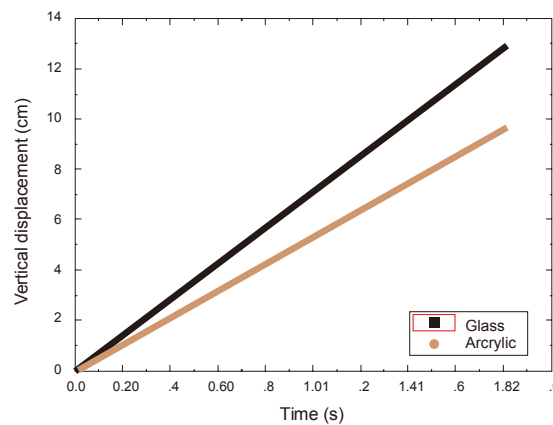
parameter that affects the V_T of a Taylor bubble. Thus V_T in [eq.7] :

$$V_T = \sqrt{\frac{\rho g R_t}{2k\mu}}$$

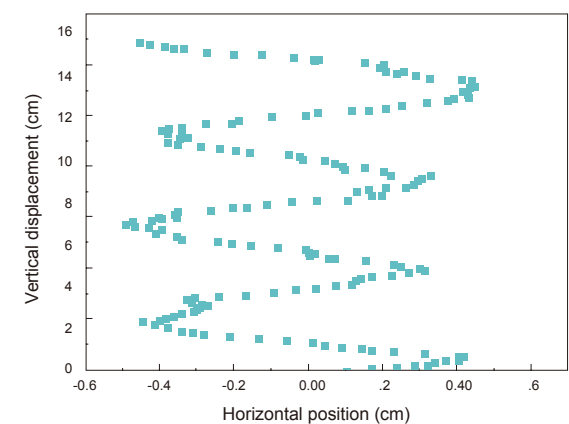
Density of glycerol and silicon oil are respectively $1.03 (\text{g}/\text{cm}^3)$ and $0.96 (\text{g}/\text{cm}^3)$. The ratio $(\mu V_T^2 / \rho)_{\text{glycerol}} / (\mu V_T^2 / \rho)_{\text{siliconoil}}$ is bigger than 0.9, which shows that the constant k in equation 6 is independent of liquid property.



[Fig.10] Effect of liquid viscosity on V_T of a Taylor bubble. The diameter of tube is 1.45cm. The tube material is acrylic. The liquid in the tube is silicon oil. The formula of fitting line is $V_T = 70\mu^{-0.5}$, where viscosity is in cP and V_T is in cm/s.



[Fig.11] Effect of tube wall property on V_T of a Taylor bubble. The tube diameter is 1.45cm. The liquid in the tube is water.



[Fig.12] Relation between the vertical and horizontal displacement of a bubble

The subsequent part investigates the effect of viscosity on V_T of a rising bubble, where equation 7. ($V_T = \sqrt{\frac{\rho g R_t}{2k\mu}}$) provides an explanation for the result.

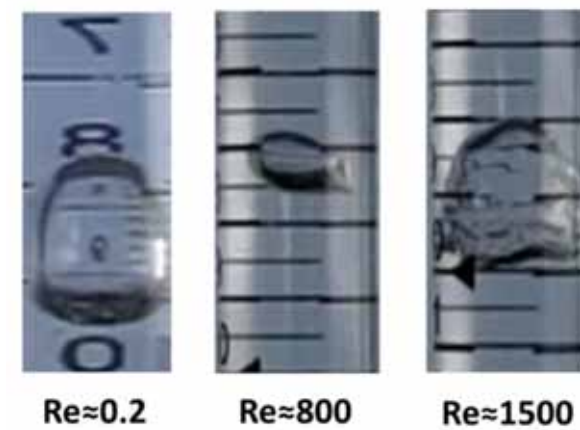
V. Effect of tube wall property on V_T of a Taylor bubble

[Fig.11] shows that the bubble in glass tube moves faster. The qualitative explanation is that due to the difference in the surface quality of the tube wall, the liquid layer between the glass tube and the bubble surface is thicker and this leads to less drag.

2. Horizontal motion of a rising bubble

Observation from the above of the tube leads to the discovery of the oscillatory horizontal motion, which is referred to as “the zigzag motion” in the references [Ref.5].

In fact, according to our observation, bubbles with smaller Reynolds number have more prominent zigzag motion. We think that the vortices behind the bubbles



[Fig.13] bubble shapes at different Re , the tube diameter is 1.45cm, the tube material is acrylic. The liquid in the tube is glycerol, water and water from left to right, respectively.

cause periodic pressure difference and thus leads to the zigzag motion. Larger Re means the liquid flow around bubble is more turbulent; thus, greater Re leads to more prominent zigzag motion.

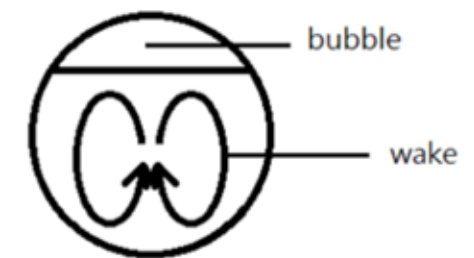
3. Shape of a rising bubble

This part investigates the shape of bubble, as shown in [Fig.12].

The observation indicates if the Reynolds number is larger, then the bubble will be more non-spherical. We think that the wake behind the bubble pushes the bubble and deforms it, just as [Fig.14] shows.

Conclusion

A bubble rising in a tube reaches its terminal velocity V_T quickly. The terminal velocity is influenced by the bubble size, the tube size, the viscosity of liquid, and the tube material. When Re is small, there is vertical motion only. When $Re > 1$, the bubble undergoes a zigzag motion in the horizontal motion and the degree of the motion is related to Reynolds number. The shape of a rising bubble depends on Reynolds number. Only when $Re \ll 1$, the bubble shape becomes more spherical.



[Fig.14] A diagram that shows the wake behind a bubble. [Ref.1] shows that the bubble becomes non-spherical when $Re > 1$.

Acknowledgements

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