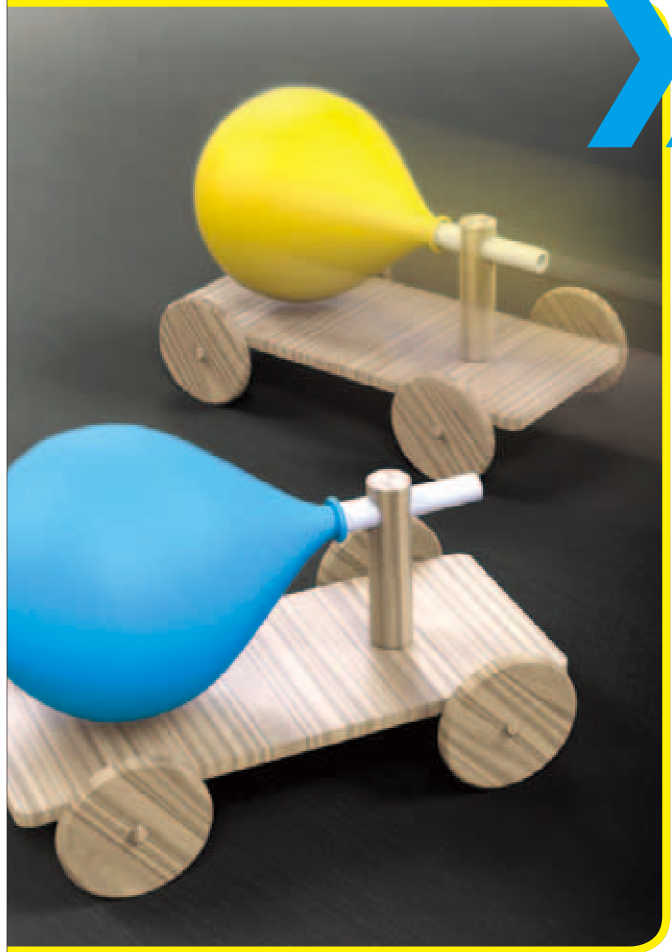
[3] Olivier Pitois et al. "Liquid Bridge between Two Moving Spheres: An Experimental Study of Viscosity Effects." Journal of colloid and interface science 231, 26-31(2000).



2010 Problem 5 : Car

Car powered by a balloon

Xinhe Yang
Nankai University, Tianjin, China
yangxinhe@mail.nankai.edu.cn

Xuewei Cao
Nankai University, Tianjin, China
xwcao@nankai.edu.cn

Abstract

This article mainly solved the distance traveled by the car which powered by a spherical balloon, an equation of relevant parameters to determine the distance was derived. In experiment part, a toy car is designed. Three variables were controlled, the radius of the balloon, the radius of the vent and the angle of the vent inclination. The distance increased exponentially with the increment of radius. There is a maximum value of distance when the radius of vent increasing. The distance decreased with the increment of the angle. The result of the experiment fitted the theory well.

Keywords

car, balloon, distance.

Introduction

A car powered by a balloon is a very attractive problem among students all around. This kind of problem is also list in IYPT before. The distance it travels depends on many parameters such as balloon, gears, wheels and so on. Many people studied the problem of the inflation, deflation and reinflation of a single balloon[2].The pressure in the balloon when inflating and deflating[3]. The damage and the healing of the balloon when inflating and deflating[4]. They also studied the difference between the spherical and cylindrical balloon[5]. The elastic instability of balloon is also discussed[6]. But few of them have analyzed the problem of the car. In order to solve the problem the car is divided into two parts, one is the power part, the other is the resistance part. We can find the balloon here is very important, because the balloon is the only thing that provides power.

Theory for car

The stress exerts on the balloon is defined by(1a), the thickness of the balloon will change (1b), where is the constant of the balloon, like Young modulus.

$$\tau = \frac{df}{ds} \quad (1a) \qquad \tau = c\frac{\Delta h}{h} \quad (1b)$$

Assuming the volume of the membrane is a constant, then (2), where r is the radius of the infinitesimal area and h is the thickness of the infinitesimal area after the balloon inflated.

$$r_0^2 h_0 = r^2 h \quad (2)$$

Regardless of the Mullins effect[6], we just make the simplest model. We do an integral on the area of infinitesimal, then

$$F = 2\pi c r^2 - 4\pi c r_0^2 + 2\pi c\frac{r_0^4}{r^2} \quad (3)$$

Because the number of infinitesimal will not change when inflating, the superficial

will be

$$S = n\pi r^2 = 4\pi R^2 \quad (4)$$

Here, R is the radius of the balloon and we find r can also stand for the radius of the balloon, there is only a difference in constant between them. Hence r is used as the radius of the balloon in the following text.

Then there is another assumption, when the balloon is deflating, the pressure all exerts on the hole of the balloon, so we can get (5) where R is the radius of the vent.

$$P = \frac{F}{\pi R^2} \quad (5)$$

When the air outflow from the vent we have law of conservation of momentum (6a) and Poiseuille formula[1] (6b), where l is the length of the vent, η is the friction coefficient of the air, F_r is the recoil force and r is the distance of the airflow to the center of the vent.

$$F_r t = mv \quad (6a) \qquad v(r) = \frac{P}{4\eta l}(R^2 - r^2) \quad (6b)$$

We can get (7) from them. Here ρ is the density of the air.

$$F_r = \frac{\pi R^6 \rho}{12\eta l}P^2 \quad (7)$$

After getting the recoil force, we can do the force analyze of the car, and get (8) easily. Where θ is the angle between the vent and ground, μ is the friction coefficient of the ground, s_1 is the distance the car travels when the balloon is deflating, s_2 is the distance the car travels by inertia, s is the total distance.

$$F_r(\cos\theta + \mu\sin\theta) - \mu mg - 6\pi\eta r\frac{ds_1}{dt} = m\frac{d^2 s_1}{dt^2} \quad (8a)$$

$$F_r(\cos\theta + \mu\sin\theta) - \mu mg - 6\pi\eta r v = m\frac{dv}{dt} \quad (8b)$$

$$v^2 = 2as_2 \quad (8c)$$

$$s = s_1 + s_2 \quad (8d)$$

To solve the (8a) and (8b) constants are needed to measure by experiments.

Pre-Experiment

Following parts are experiments. Iron hoops are used to control the radius of the balloon, to avoid Mullins effect[7], one balloon can only be used no more than 4 or 5 times. In experiments, we can find this influence is not obvious. The weight of the toy car is 34.23g. We neglect the weight changed by inflating a balloon and different kinds of vents, they are all small. Every experiment is recorded by camera. In video we read the distance traveled by the car to get two constants and then we can calculate 8(a). Figure 1 is the picture of the car.

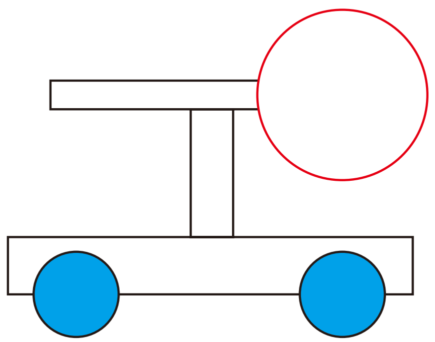


Fig. 1: Design of the car

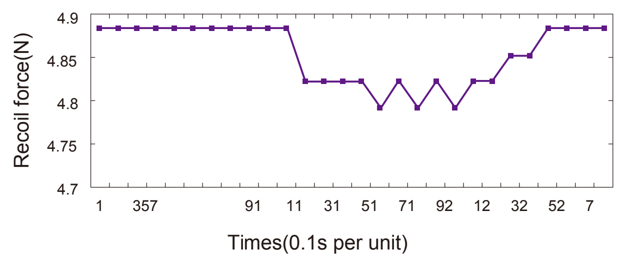
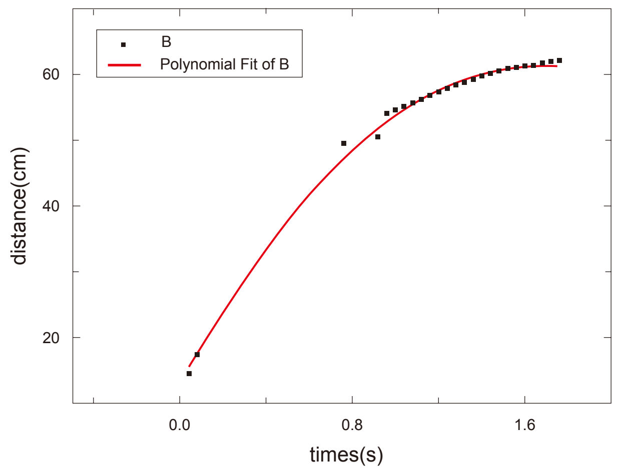


Fig. 2: Recoil force change with time
(radius of vent is 0.5cm, radius of balloon is 6.75cm)

But before the experiments on controlling variables, some pre-experiments are necessary for the experiment. First, to get the constant c of this kind of balloon, a force sensor is used to measure the recoil force of the balloon (Figure 2). We repeat the experiment with different radius of vent (Table 1). From the figure, we know that the c of this kind of balloon is 0.029N/m^2.

Table1: The recoil force changed with radius of vent
(radius of balloon is 6.75cm)

R(radius of vent)(cm)	0.4	0.5	0.675
F(Maximum force)(N)	1.068	1.221	2.137



times(s)

Model	Polynomial		
Adj. R-Square	0.99677		
		ValueS	tandard Error
B	Intercept	12.99906	0.50693
B	B1	58.38586	1.11222
B	B2	-17.6704	0.59259

Fig. 3: Distance the car travel change with time& the fitting table
(push it slightly to measure the friction of ground)

The friction of the ground was estimated by pushing the car slightly and recording it by camera. We read the position of the car through the video and use Origin to fit the quadratic curve (Figure 3). The acceleration of the car is the second derivative of the curve. The coefficient of friction μ is 0.036 from the curve.

Experiment and discussion

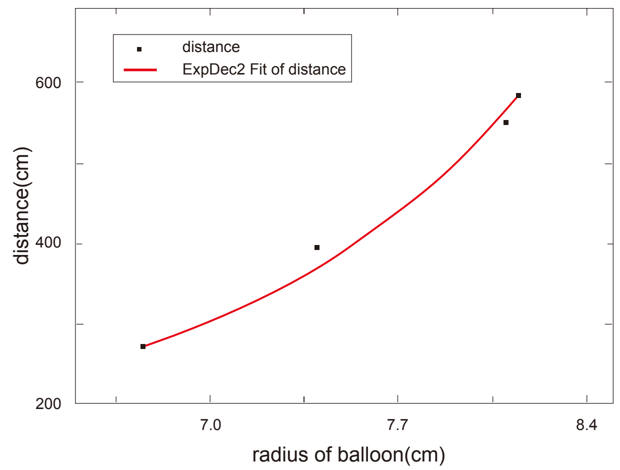


Fig. 4: Distance the car travels change with radius of balloon
(radius of vent is 0.5cm, angle is zero)

Three variables are controlled in experiments. First is the radius of the balloon. (Figure 4, Table 2) We can find that the distance the car travels increase exponentially with the radius of balloon. The data is fitted by the Origin.

Table 2: Distance the car travels change with radius of car
(radius of vent is 0.5cm, angle is zero)

r(radius of balloon)(cm)	6.75	7.4	8.1	8.15
s(distance)(cm)	272	393	552	586

Second is the radius of vent, for this, we used different kinds of tubularis, because Poiseuille formula can be used in different kinds of vent. (Table 3 the angle is 0°, the radius of balloon is 6.75cm)

Table 3: Distance the car travels change with radius of vent
(radius of balloon is 6.75cm, angle is zero)

R(radius of vent)(cm)	0.4	0.45	0.5	0.675
S(distance)(cm)	184	314	272	240

We can know that there is a best radius of vent from the table, in our experiment the radius is 0.45cm. We can easily understand it. If the vent is too small, though the recoil force will last for a long time, the force is small. If the vent is too large, the recoil force can't last long, the distance is still short. So there is a maximum in the experiment. Because the limit of our experiment, we can only find the best radius is in between 0.4-0.5cm.

The third variable is the angle of the vent. If we inclined the vent a little, the recoil force will have a vertical component, so the friction of the car will decrease, it may go farther. If the angle is too large, the horizontal component will decrease a lot, the distance will decrease. So there may be a best angle.(table 4 the radius of vent is 0.5cm, the radius of balloon is 6.75cm)

Table 4: Distance the car travels change with angle of vent
(radius of vent is 0.5cm radius of balloon is 6.75cm)

Angle of the vent(°)	0	10	20	30
Distance(m)	2.72	2.22	2.05	1.86

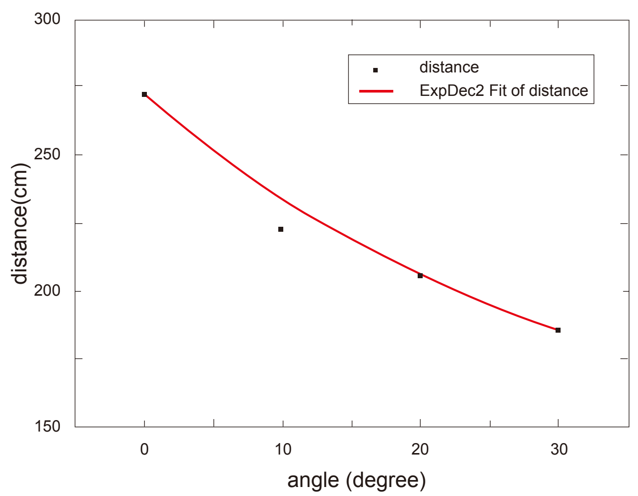


Fig. 5: Distance the car trave change with angle(by experiment)
(radius of balloon is 6.75cm, radius of vent is 0.5cm)

From the experiment we find the distance decrease when increasing angle, (Figure 5) the best angle here is zero. Because we know the friction of the ground is so small, so the best angle is small and it can't improve the result obviously. We use maple to draw the image of function cosx+μsinx where 0<x<30° (Figure 6) μ is 0.036 here. From Figure 6 we can find that the value only increases a little in a small angle about 5° and the increment can even be neglected. So the result of experiment is obvious to know.

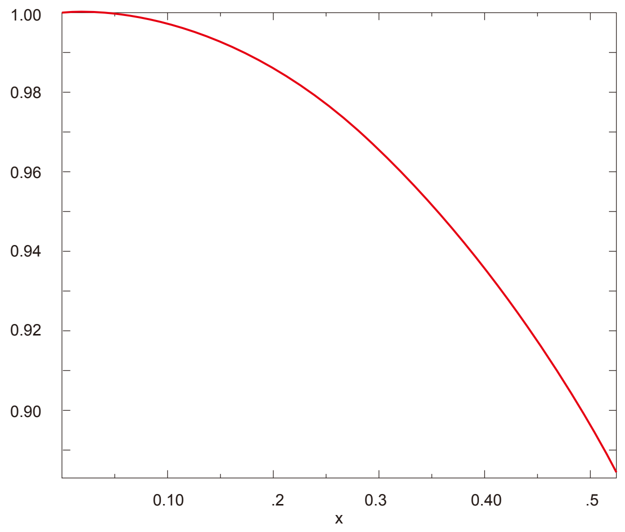


Fig. 6: value of cosx+□sinx change with x(by maple)

Conclusion

In this article, the force inside of a balloon was derived. A equation that can predict the distance was also derived, but it's really a pity that the theory do not fit very well when the radius of balloon is large than 8cm (the theory predict it can go about 9 to 11 meters, but the experiment shows it only 5 to 6 meters.) and the error will be larger if the radius of balloon increases. the reason may lie here, first the Mullins effect was neglected here, we just use the simplest model of balloon. Second, we assumed if there is a hole in balloon, the force inside will all exert on it, in fact, there is energy loss in balloon when deflating and energy loss in sphere balloon is larger than in cylindrical balloon. Moreover, energy loss will increase with the radius of balloon. In experiment, we controlled three variables. Our theory can predict the result in some degree. The best radius of vent can be very useful in making balloon car. In addition, the gear, radius of wheel, length of axle, shape of car can all influence the distance.

Acknowledgement

I really need to thank Heng Zhao, we worked on this problem in different way, but he helped me a lot on experiments and simulated the function. I also need to thank Teacher Xuewei Cao, he gave me the experiment room. I also need to thank teacher Chuanyong Li, who gave me the money to finish my experiments, and the teammates for TCPT, they helped me in different ways.

Reference

[1] Zhao Kaihua & Luo Weiyin .Li Xue. China(2004).

[2] Ingo Müller, Henning Struchtrup. Inflating a Rubber Balloon. Mathematics and Mechanics of Solids 2002 7: 569(2002).

[3] D. R. Merritt and F. Weinhaus. The pressure curve of a rubber balloon. Am. J. Phys. Vol. 46, No. 10(1977).

[4] Domenico De Tommasi, Salvatore Marzano, Giuseppe Puglisi, Giuseppe Zurlo. Damage and healing effects in inflated rubber balloons.

[5] Landon M. Kanner, Cornelius O. Horgan.Elastic instabilities for strain-stiffening rubber-like spherical and cylindrical thin shells under inflation. International Journal of Non-Linear Mechanics(2006).

[6] Ren Jiusheng. Elastic Instability of Pseudo-Elastic Rubber Balloons. CMC, vol.7, no.1(2008).

[7] Julie Diani , Bruno Fayolle, Pierre Gilormini.(2009). A review on the Mullins effect. European Polymer Journal 45 601–612(2009) .