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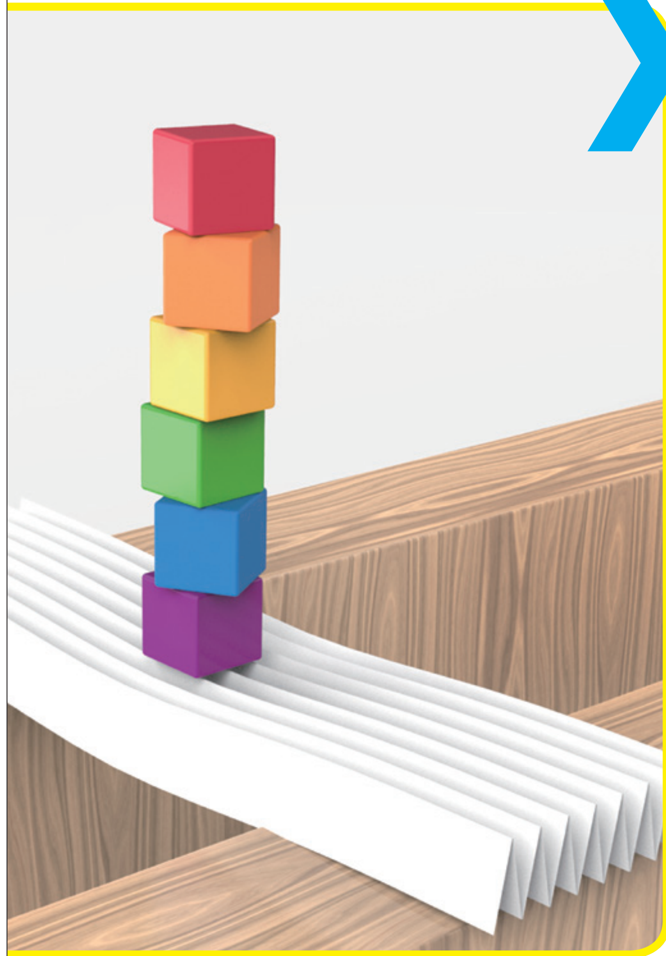

# 2013 Problem 1 : Invent Yourself

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## Abstract

This is the first problem of 2012 IYPT, it's a dynamics problem. This article analysis the problem both in theory and experiments.

## Keywords

*paper bridge 280mm strength*

## Introduction

The problem is: It is more difficult to bend a paper sheet when it is folded in "accordion style" or rolled into a tube. Using a single A4 sheet and a small amount of glue, if required, construct a bridge spanning a gap of 280 mm. Introduce parameters to describe the strength of your bridge, and optimise some or all of them. The key words of the problem include: A4 paper, a small amount of glue, and 280mm gap. The problem asks us to build a certain kind of paper bridge and use parameters to describe the strength of the bridge and optimize it.

## Theory

First look at the potential parameters. The size of the paper is fixed to A4, but the type of the paper can vary. Of course, the stronger the paper is, the stronger the bridge is ( Fig. 1 and Fig. 2). For the glue, we need to choose the glue which can stick well and is not fragile.

Fig.1 This is a bridge in printing paper, it can be damaged easily with one weight.

Fig.2 This is a bridge in copperplate paper, it is stronger that printing paper, so it is in good condition under one weight.

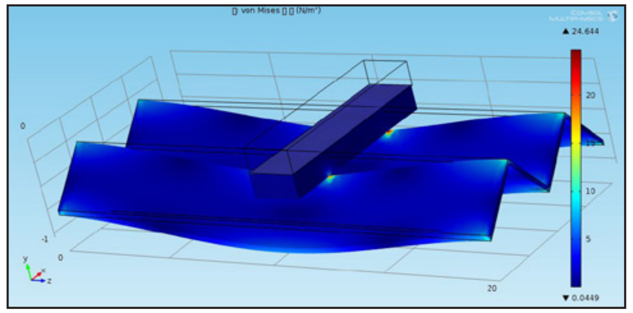

Fig. 3 The simulation based on "Commsol", to analyze the weakest part of the accordion style bridge.

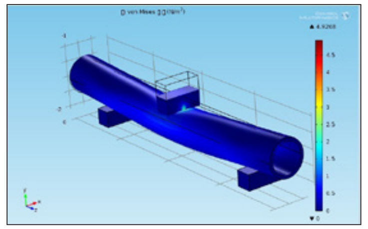

Fig.3b

We will first define load-bearing capacity. The definition is that the maximum load the bridge can bear within a limited load-bearing area before any irreversible deformation occurs. Here, irreversible deformation can be judged in the two ways. The first through observation, that we can see the occurrence of bridge deformation. The second one is through failure of load-bearing of the bridge. There is a critical value for a certain bridge, and when the mass of the load surpasses that critical value, the bridge fails to continue bearing the load, and it collapses. That is, we can both see a bridge collapse through observation and demonstrate the bridge failure through calculation of load-bearing capacity. Later I will prove this with an experiment.

For the strength of the bridge, we believe that the

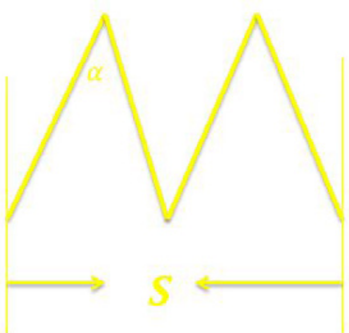


Fig. 4 The simulation based on "Commsol", to analyze the weakest part of the tube style bridge.

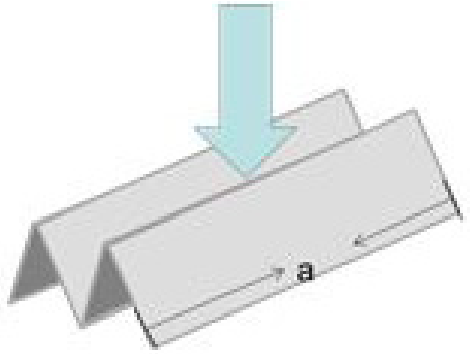


Fig. 5 The sectional view of the accordion style bridge.

load bearing capacity of the weakest part of the bridge represents the load bearing capacity of the bridge. So we did simulation to pinpoint the weakest part. The simulation was based on the software "Commosl", and the color bar represents the relative amount of the stress (Fig.3). This simulation also shows the weakest part of an accordion style bridge, which is in the middle. Fig.4 shows that in a tube bridge, the weakest par is also in the middle. The reason why we need to identify the weakest part is that damage to part of a bridge destroys the whole bridge. Therefore we determined the condition of the weakest part represents the condition of the whole bridge.

After locating the weakest part of the paper, we then identify parameters.

The parameters for an accordion style bridge include: the number of pleats(one pleats means one peak, up and down) N, and the span S.(see Fig. 5)

For accordion style, there were two kinds of damages. The first one was collapse. When the pleats were not too

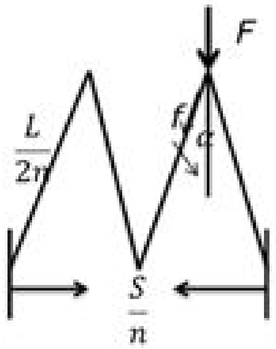


Fig. 6 The diagram for force analysis.

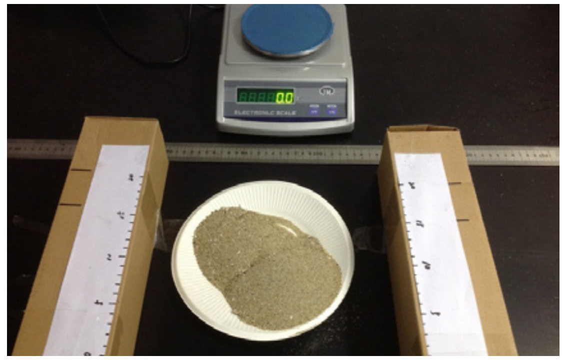

Fig. 7a The sectional view for analyzing the accordion style bridge.

high, the bridge will crashed in the middle. The second one was warping. When the pleats are fairly high, each pleat warped in the middle. The two damages above are different in its orientation of collapse. For a bridge to cpllapse, in a certain broadside, there is a force "$f$" going down( Fig. 6 and Fig. 7, here we deem the force $F$ is the total force on the pleats, so $2f\cos\frac{\alpha}{2}=F$ (1)

So the torque is $\frac{fa}{4}$, and there is a layer in the middle of the beam which is not stretched nor compressed. We choose the origin O in this layer, we can get the length at any z (z is the vertical scale in the coordinate we choose), which becomes (reference 1):

$$\theta(R-Z)={}^{l(R-z)}/_{R} \qquad (2)$$

For the "R" is the radius of curvature of the beam, and the $\theta$ is the radius angle of the beam, "$l$" is the length of a plain A4 sheet, which can be understood in the Fig.7.

And $\Delta l=-lz/R$, so we can obtain the strain $\epsilon=\frac{\Delta l}{l}=-\frac{Z}{R}$(3).The area at $dz$ is: $dS=\tau d$ (4).

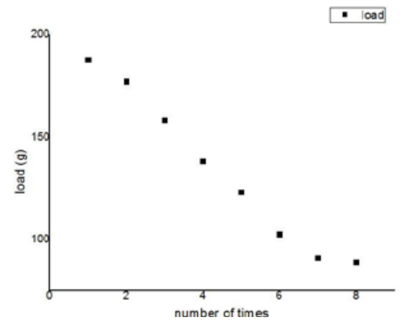


Fig. 8 Pictures for apparatus.

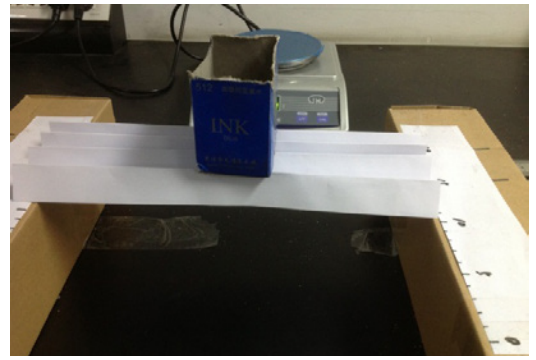

fig.7b

According to Hooker's law, the stress is:

$$f=-\frac{Yz}{R} \qquad (5)$$

$$df=-Y\tau z dz/R \qquad (6)$$

and the torque of couple to the origin O is: $dM=zdf=-Y\tau z^2 dz/R$(7), so the total torque is:

$$M_{inner}=\int dM=-\frac{Y\tau\left(\frac{L}{2n}\right)3}{12R} \qquad (8)$$

When the bridge collapse, it means the curvature $k$ of the bridge is above a certain value, so we deem the curvature to be $k$, and

$$K=\frac{1}{R}=\frac{12M_{out}}{Y\tau\left(\frac{l}{2n}\right)3}=\frac{3Fa\cos\alpha}{2y\tau\left(\frac{l}{2n}\right)3} \qquad (9)$$

$$\cos\alpha=\frac{\sqrt{(\frac{l}{2n})^2-(\frac{s}{2n})^2}}{\frac{l}{2n}} \qquad (10)$$

With the total force $F_{total}=nF,\ F_{total}=\frac{2nkY\tau\left(\frac{L}{2n}\right)^3}{3a\sqrt{\left(\frac{l}{2n}\right)^2-\left(\frac{s}{2n}\right)^2}}$ (11),we

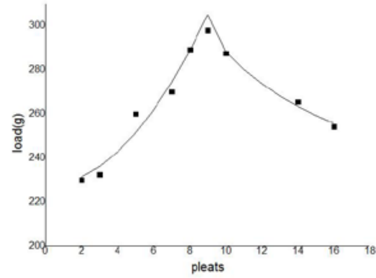


Fig. 9 The diagram for force analysis.

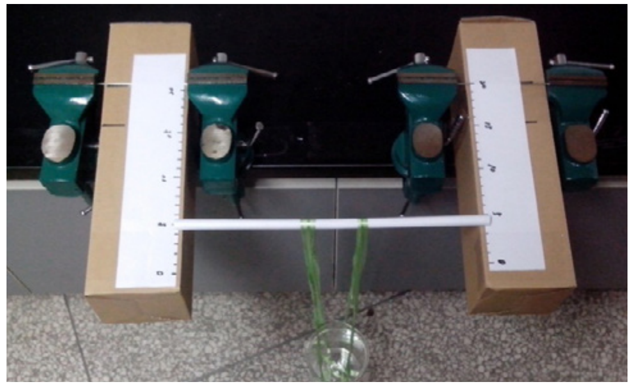

Fig. 10 Scatter diagram to show the existence of critical value.

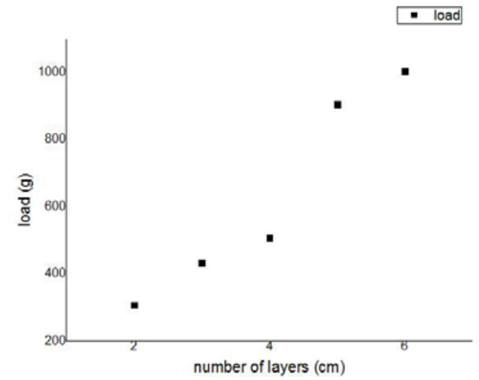


Fig. 11 Tendency chart to get the best "N"

can see as $F_{total} \propto \frac{1}{n}$.For the warping style, the force works similarly to the bridge, which is: $F\Delta x$ . If the work surpasses the potential energy that the bridge store, the bridge can store, we obtain these equation:

$$\Delta E_{ep} = \int_0^{\theta} M_{inner} d\theta \quad (12)$$

$$\therefore \quad \frac{\theta^2 Y a \tau 3}{12l/n} \leq \frac{F\Delta X}{2} \quad (13)$$

And:$F_{total} = nF = \frac{Ya\tau^3 n2\theta 3}{6L\Delta X}$ (14), here we have neglected the gravity of the paper.For the tube style, The parameters are diameter *d*, and the thicknessτ( reference 2).

## Experiment Setup

The apparatus for the experiments are shown in Fig.8 We used the sand as the load, and after it collapseected, we collected the sand, and valued it with

an electronic balance. (Fig. 9)

We used a paper box to hold the sand. The floor space of this box is:4.5×8 $cm^2$

The first experiment proves that critical value exists. We used one accordion style bridge with 9 pleats and loaded sands on it for 8 times, collected the data(see Table.1):

Table 1. Use one single bridge for 8 times, record the load

| Times | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Load(g) | 187.9 | 177.2 | 158.3 | 138.4 | 123.3 | 102.7 | 91.2 | 89.0 |

We can see from Fig. 10, that after irreversible deformation, the loading capacity decreases, so there is a critical value.

Table 2. Build bridges with different number of pleats

| *N* | 2 | 3 | 5 | 7 | 8 | 9 | 10 | 14 | 16 |
|---|---|---|---|---|---|---|---|---|---|
| Load(g) | 230.3 | 232.5 | 262.1 | 274.7 | 289.2 | 297.8 | 287.4 | 265.4 | 254.1 |

Then we optimized the parameters experiments. The parameters include: number of pleats N(see data in Table 2),

Table 3. Build bridges with different length of span

| Span *S* (cm) | 11.0 | 9.00 | 8.01 | 7.50 | 5.50 | 4.00 | 3.00 |
|---|---|---|---|---|---|---|---|
| Load (g) | 193.4 | 248.8 | 261.6 | 297.8 | 281.1 | 267.2 | 253.9 |

and the span *S*( Table 3 and Fig. 12):

So the best *N* is 9, and the best span is 7.50 cm.

So we conducted an experiment to optimize the tube. ( Fig. 13 )

For the thickness, we kept the diameter to be the same 1.50 cm so that the layers of the tube stood for thickness.

From Fig. 14, we can see the thicker the bridge is, the stronger it becomes.

TTable 4. Test the load of the tube of different diameter

| *d(cm)* | 1.17 | 1.87 | 2.80 | 3.13 | 6.47 |
|---|---|---|---|---|---|
| Load(g) | 176.2 | 191.0 | 183.7 | 181.9 | 161.3 |





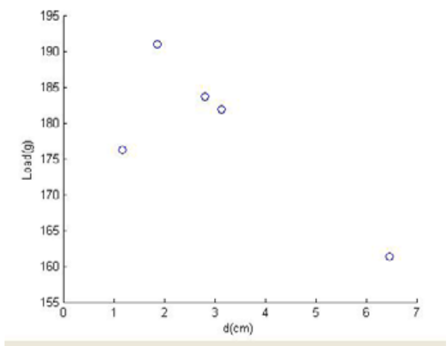


Fig. 12 Tendency chart to get the best "S".

Fig. 13 Pictures for the load bearing experiments for tube bridges.

For the diameter, the data is in Table 4. In this group of experiments, the layer of the bridge was one, which meant the thickness of the bridges were the same value. The final data is as Fig. 15 shows, so the best diameter is 1.87cm.

## Results and Discussion

For the experiment, we did some error analysis.

1. For the accordion style, the folding was not uniform. The bridges were hand-made, the width of each bridge differed slightly. If we suppose Length of pleat as b, the unevenness as =as $= \frac{bmax - b_{min}}{bmax + b_{min}}$ (15), the bigger it is, the weaker the bridge becomes;
2. The stress was not uniform, for we can't make the sand equipartition;
3. The density was not uniform, for the density of paper varied in different parts in the process of paper manufacture, and thus unavoidable.
4. The way of damage was not ideal: the Young modulus of bridge might change.
5. The differences in paper, we cannot find the same paper in the world.

The problem is "invent yourself", so inventing is important.

We can think of other bridge style such as plane, cone, triangular tube, rectangular tube, and solid tube and so on---However the style changes, the theory remains the same. We need to separate the loading force and make the component force the stretch or compress force.

We can also combine several tubes together. For example, we can use glue to stick several tube together. (Fig.16)

## Conclusion

The conclusion is: for accordion style, the best situation is the pleats are 9 and the span is 7.50 cm, at which time it can bear 297.8g sands. For the tube, the thicker, the better. The best diameter is 7.5cm , and it can bear 191.0g.

## Acknowledgement

This work is supported by National Science Fund for Talent Training in Basic Sciences (No.J1103208).

I cannot thank professor Cao xuewei and Yuhua enough for their instruction on this paper. And thanks my teammates who helped me solve this problem.

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