2018 Problem 6: Ring Oiler

Periodic Motion of the Ring in a Rotating Shaft

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Abstract

An oiled horizontal cylindrical shaft rotates around its axis at constant speed. Make a ring from a cardboard disc with the inner diameter roughly twice the diameter of the shaft and put the ring on the shaft. Depending the tilt of the ring, it can travel along the shaft in either direction. The phenomena described above are studied in this paper. Combine the Euler's dynamics and kinematic equations with the generalized coordinates derived from analyzing constrain of the ring, the relationship between the displacement and angular velocity of ring with time is researched.

1 Introduction

In 20th century, ring oiler is used to lubricate bearing. In car, the rotation of the shaft is often used to drive a ring partially immersed in lubricating oil, and then the rotation of the ring is used to bring the lubricant to the shaft, so as to achieve the lubrication of the shaft. In the laboratory, we built a simple device to observe the motion of the ring on the oil-coated shaft. When a ring is placed on a rotating oiled horizontal cylindrical shaft, it can move in any direction along the shaft due to the tilt of the ring. Through observation, we also find that the ring and the shaft are in point contact, and the ring can reciprocate on the shaft. The periodic motion of a ring on a shaft is a very interesting phenomenon.

However, few literatures have studied this phenomenon in depth. In this paper, Euler dynamic and kinematic equations are employed to investigate this phenomenon according to Euler rigid body. A clear dynamic equation of ring motion is obtained.

2 Theory

Establish the physical model shown in Figure 1 and satisfy the following assumptions.

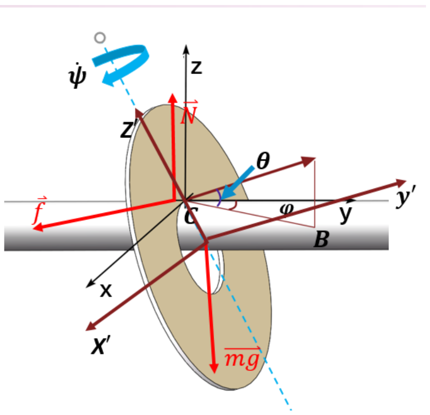

FIG. 1. A schematic diagram of the generalized coordinates of the motion of the ring on the shaft. x, y, z are the coordinates of fixed reference frame. x, y, z are the coordinates of the moving reference frame. So, we see the ring as a rigid body. Precession angle, φ, is the angle between the projection of the x-axis in the xy plane and the x-axis. Nutation angle, θ, is the angle between the y-axis and the y-axis. Self rotation angle, ψ, is the angle between the x-axis and the intersection of xy plane and xy plane.

1. The shaft is an ideal cylinder. The ring is an ideal rigid body;

2. The ring's mass is evenly distributed;

3. The vibration of motor is ignored and the shaft rotates smoothly.

Because of the rotation of the shaft, the ring placed on the shaft will rotate due to friction. During the rotation of the ring, the perturbation will deflect the ring and cause the motion of the ring parallel to the shaft. Under the combined action of frictional force and ring speed, the ring will reciprocate in the transverse direction of the shaft.

Based on Euler's rigid body theory, the general motion of rigid body has six degrees of freedom: three are rotational degrees of freedom and three are translational degrees of freedom. From the observation of experimental phenomena, it is found that there is always a contact point between the ring and shaft when ring moves on the horizontal shaft. Therefore, the degree of freedom of the ring motion in the model of Figure 1 is reduced to five, that is to say, five generalized coordinates are needed to describe it. In Figure 1, a fixed reference frame, that is terrestrial reference frame, and a follow-up reference frame are established, which are represented by x y z and x' y' z', respectively. The corresponding Euler angle θ φ ψ are defined as three generalized coordinates. The fourth generalized coordinate α is defined as the contact position between the ring and the shaft, as shown in Figure 2. The last generalized coordinate is defined as the

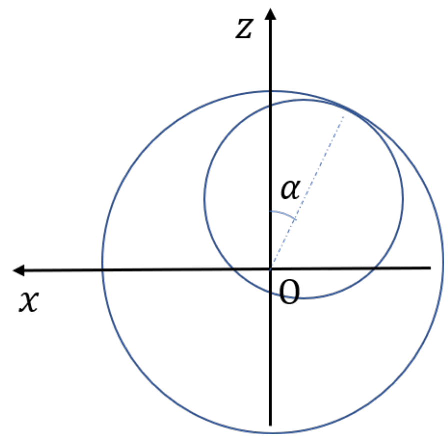

FIG. 2. A schematic diagram of the contact point between the ring and the shaft as a generalized coordinate. It is side view. The big circle represent the inner circle of ring. The little circle represent the shift. It is the angle between the line connecting of two center and the vertical direction.

displacement in the y direction of the ring moving on the shaft. [1]

According to Euler's dynamic equations

$$\begin{cases} I_1\dot{\omega}_x-(I_2-I_3)\omega_y\omega_z=M_x \\ I_2\dot{\omega}_y-(I_3-I_1)\omega_z\omega_x=M_y \\ I_3\dot{\omega}_z-(I_1-I_2)\omega_x\omega_y=M_z \end{cases} \tag{1}$$

where I is the moment of inertial around x', y', z' of the ring in the moving reference frame, M is the total torque of the ring, ω is angular velocity's component around x', y', z' of the ring in the moving reference frame.

The moment of inertia I around the x, y and z axis can be expressed as:

$$\begin{cases} I_1=\frac{1}{4}m(R^2+R'^2)+\frac{1}{12}md^2 \\ I_2=\frac{1}{2}m(R^2+R'^2) \\ I_3=\frac{1}{4}m(R^2+R'^2)+\frac{1}{12}md^2 \end{cases} \tag{2}$$

where m is ring's mass. R and R' are inner and outer radius of ring, respectively. d is thickness of ring. ω_0 is rotating velocity of shaft. Because the ring is symmetrical, the moment of inertia around the x-axis is the same as that around the z-axis.

According to Euler's kinematic equations, angular velocity ω can be expressed as [1]:

$$\begin{cases} \omega_x=\dot{\psi}\sin\theta\sin\varphi+\dot{\theta}\cos\varphi \\ \omega_y=\dot{\varphi}\sin\theta\cos\varphi-\dot{\theta}\sin\varphi \\ \omega_z=\dot{\varphi}\cos\theta+\dot{\psi} \end{cases} \tag{3}$$

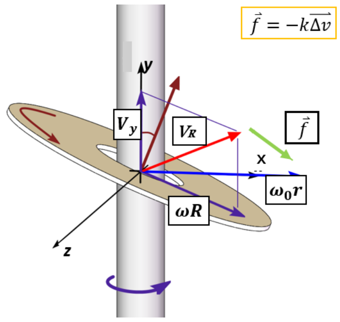

FIG. 3. Diagram of dynamic analysis of ring motion on shaft. v_y is the component of the ring in the y direction. $R\omega$ is the linear velocity of the ring rotating along the axis of symmetry. $\omega_0 r$ is the shaft rotation speed. v_R is the total velocity of the ring. $\vec{f}$ is the wet friction caused by the velocity difference.

For torque M, it can be expressed as

$$\begin{cases} M_x = -f_y R\cos\theta\cos\alpha - f_z R\cos\theta \\ \qquad -N\left(R\sin\theta\cos\varphi \pm \dfrac{d}{2}\cos\varphi\right)\cos\alpha \\ M_y = -f_x(-R\cos\theta\cos\alpha) + Rf_z\sin\alpha \\ \qquad -NR\sin\alpha\cos\theta \\ M_z = f_x\left(R\sin\theta\cos\varphi \pm \dfrac{d}{2}\cos\varphi\right) - f_y R\sin\alpha \\ \qquad -N\sin\alpha\left(R\sin\theta\cos\varphi \pm \dfrac{d}{2}\cos\varphi\right) \end{cases} \tag{4}$$

where f is the wet friction force between the ring and the shaft. Wet friction force f causes the rotation of the ring when the shaft rotates, which can be determined by

$$\vec{f} = -k\overrightarrow{\Delta v} \tag{5}$$

$\vec{f}$ is the total friction of the ring. k is the coefficient of wet friction, which is a constant for the same situation. $\overrightarrow{\Delta v}$ is the velocity difference between the ring and the shaft at the contact point. It means that the wet friction is proportional to the velocity difference.

In the x and y directions, wet friction force f can be expressed as

$$\begin{cases} f_x = -k(-\omega_0 r + \omega_y R\cos\varphi) \\ f_y = -k(\omega_y R\sin\varphi + \dot{y}) \end{cases} \tag{6}$$

In Figure 3, the wet friction force and the direction of motion of the ring and shaft are shown. The composition of velocity law of v_y and ωR is led into the velocity of the contact point in the ring v_R.

According to centroid motion theorem,

$$\begin{cases} m\ddot{x} = f_x - N\sin\alpha - k\dot{x} \\ m\ddot{y} = f_y \end{cases} \tag{7}$$

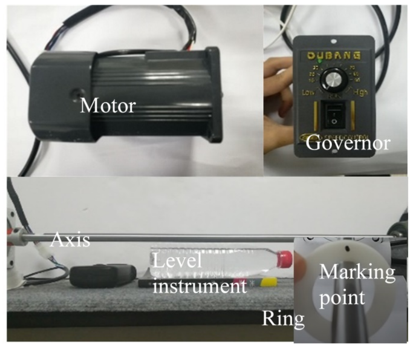

FIG. 4. Image of the experimental device for the motion of the ring on the shaft.

Euler's dynamic equations and centroid motion theorem can completely describe the motion of rings. However, the analytical solutions of these equations are almost impossible to obtain. Fortunately, we can obtain the numerical solutions of the relationships between some parameters with the aid of experiments.

3 Experiment and discussion

An experimental device is built as shown in Figure 4. An adjustable speed motor causes the shaft to start turning. Driven by the motor, the shaft begins to rotate, and the wet friction between the shaft and the ring causes the ring to rotate. When the ring begins to rotate, it immediately tilts and then moves back and forth on the shaft. A high-speed camera is used to record the motion of the ring. Mark a black dot on a white ring to facilitate subsequent video processing by Tracker software. In the experiment, the relationship between the angular velocity and the displacement of the ring and time on the shaft were researched.

In order to compare the experimental results with those of theoretical analysis, some parameters need to be determined through experiments in order to solve the theoretical analysis equations nu-

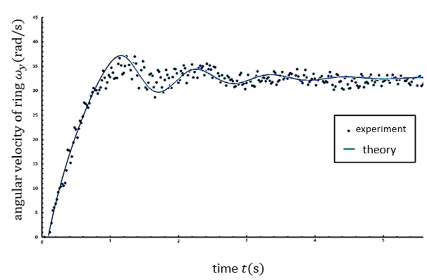

FIG. 5. The relationship between the angular velocity and the time of the ring rotation.

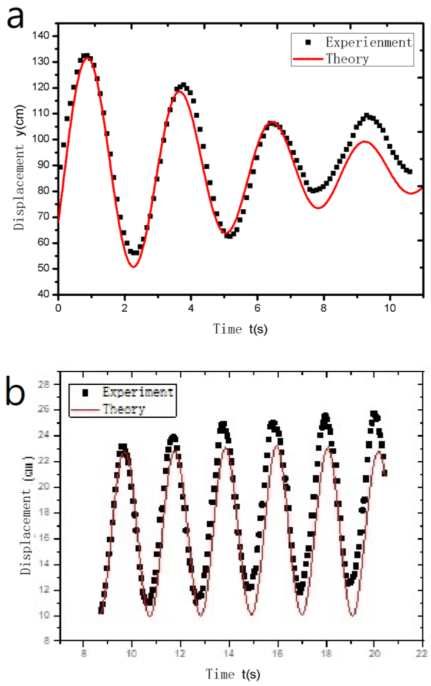

FIG. 6. The relationship between the displacement of the ring and time on the shaft.

merically by Mathematica software. The wet friction force f can be determined by experiments in the theoretical analysis equations. The expression of Newton's second law of motion of wet friction force is integrated. The expression of wet friction factor k can be obtained as follows.

$$\frac{d(\omega R)}{dt} = -(\omega R - \omega_0 r) \tag{8}$$

Then we can integral this equation.

$$k = \frac{I_y}{R^2 t} \ln \frac{\omega_0 r}{\omega_0 r - R\omega} \tag{9}$$

I_y is the moment of inertia around y-axis. R is the inner radius of the ring. r is the radius of shift. ω_0 is the angular velocity of the shift. We measured the angular velocity in the time t from 0 to ω by Shooting with a high speed camera. By the measurements and calculations, wet friction factor is approximately 0.40 Ns/m. Based on this, we can numerically solve the equations obtained from theoretical analysis, and then compare the experimental results.

The relationship between the angular velocity and the time of the ring rotation is shown in Figure 5. The angular velocity of the ring increases gradually, and then fluctuates near a certain speed. Finally, the angular velocity of the ring is stabilized. The curves obtained by the numerical solution of the theoretical analysis equation are in good agreement with the experimental data points.

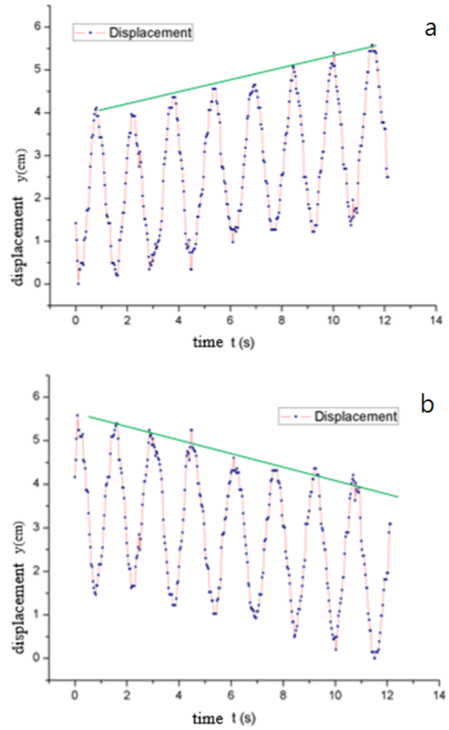

FIG. 7. The relationship between the displacement of the positive and negative sides of the ring on the shaft and the time.

Figure 6 shows the relationship between the displacement of the ring and time on the shaft. The ring will first take reciprocating motion with a larger displacement on the shaft, then the maximum displacement of the reciprocating motion will decrease rapidly, and finally the ring will move steadily to and fro with a smaller displacement. The theoretical numerical simulation curve is in good agreement with the experimental data points. It is noteworthy that the deviation between the measured data points and the theoretical simulation curve increases gradually with time when the ring starts to steadily reciprocating motion. The experimental data show that the equilibrium position of the ring in steadily reciprocating motion is offset in a certain direction. It is assumed that this is due to the uneven distribution of the mass of the ring. In order to verify this hypothesis, we did the above experiment again with the ring flip, and the corresponding experimental results are shown in Figure 7. When the ring is turned, the equilibrium position of the ring motion is shifted in another direction. Therefore, when the ring moves steadily to and fro on the shaft, the change of equilibrium position is caused by the mass distribution of the ring itself.

4 Conclusion

The wet friction between the ring and the shaft causes the ring to rotate on the shaft and to reciprocate in the transverse direction of the shaft. The motion of a ring on a shaft is theoretically analyzed by using Euler rigid body theory. The relationship between the angular velocity and the displacement of the ring and time on the shaft are explored experimentally. The angular velocity of the ring rotating on the shaft increases gradually and remains stable at last. When the ring moves horizontally to and fro on the shaft, the amplitude decreases rapidly and then becomes a certain value. The results of theoretical numerical simulation are in good agreement with the experimental results.

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References

[1] Shangnian Jin. and Yongli Ma.. (2002). Theoretical mechanics, 100-131. Higher Education Press