2012 Problem 9: Magnet and Coin

A magnetic coin's motion under a magnetic field

Abstract

We studied the motion of magnetic coins under a magnetic field, and investigated the key reasons that affected the critical angle $\theta_{crit}$. When below or above $\theta_{crit}$, the coin would reversed or fell down respectively. The strength of magnetic field, gradient of magnetic field, and distribution of magnetic domain are shown to be crucial to $\theta_{crit}$ value. We also find that the $\theta_{crit}$ has a maximum about 35°. Some theoretical calculations are also included for the explanation of experimental results.

Introduction

What is a magnetic coin? A magnetic coin is a metallic disc which contains magnetic substance, for example iron, nickel, and cobalt. When we applied a magnetic field, the coin would be magnetized. Well aligned magnetic moment would be attracted or repelled by magnetic field.

Figure 1 shows the magnetic coin used in our experiment. It can be fully magnetized when the magnetic field is large enough (1000–2000 Oe). There is no remanent magnetization when $H=0$ Oe.

We study the motion of coins in the external magnetic field and discuss why the coin falls down or reverts. $\theta$ is the angle between the coin and the magnet surface normal, as shown in Figure 2(a). When the angle $\theta$ is larger than $\theta_{crit}$ (critical angle), the coin will fall down, otherwise it will revert. (Figure 2(b) and 2(c)).

Figure 2. (a) The illustration of the angle $\theta$. (b) The coin falls down when $\theta > \theta_{crit}$. (c) The coin reverts when $\theta < \theta_{crit}$.

Figure 3(a) shows the experimental setup. We used a straw to push the coin quasi-statically. In order to study the variables that affect the critical angle, we change three parameters, which are (1) strength of magnetic field, (2) Aspect ratio of coin, and (3) distance between coin and magnet. We used a Gauss meter to measure the magnetic field of the magnet.

In Experiment (1), as shown in Figure 4, we used two kinds of magnets: (i) Strong magnet, $H=3000$ Oe or $5000$ Oe and (ii) Weak magnet. We turned the $H$ by number of magnet. The $\theta_{crit}$ increase from 13° to 35°.

In Experiment (2), as shown in Figure 5, we used two kinds of coin. One is made of pure steel. The other is made of ninety-five percent steel and five percent nickel. Each coin has different shape. The different shape means that coins have different aspect ratio $k$,

\[ k = \frac{L}{D} \]

$L$ is the diameter of coin. $D$ is the thickness of coin.

In Experiment (3), we changed the distance between coin and magnet, in order to change the magnetic field distribution, and gradient of magnetic field. We observed the corresponding changes of $\theta_{crit}$.

Figure 3. (a) The picture of the experimental setup. The coin is HK ten dollar. (b) The yellow crossbar is the sensor of Gauss meter which is used for the measurement of magnetic field.

Experimental Result

Exp. (1) Strength of magnetic field

In Experiment (1), as shown in Figure 4, we used two kinds of magnets: (i) Strong magnet, $H=3000$ Oe or $5000$ Oe and (ii) Weak magnet. We turned the $H$ by number of magnet. The $\theta_{crit}$ increase from 13° to 35°.

In Experiment (2), as shown in Figure 5, we used two kinds of coin. One is made of pure steel. The other is made of ninety-five percent steel and five percent nickel. Each coin has different shape. The different shape means that coins have different aspect ratio $k$, which is the ratio of the diameter to the thickness.

\[ k = \frac{L}{D} \]

$L$ is the diameter of coin. $D$ is the thickness of coin.

In Experiment (3), we changed the distance between coin and magnet, in order to change the magnetic field distribution, and gradient of magnetic field. We observed the corresponding changes of $\theta_{crit}$.
In Figure 4, the red dots represent the magnet of type A, yellow dot is type B, and blue dot is type C. The magnet of type A is the weakest magnet, and we used different number of magnet from one to eleven to increase the magnetic field. From Figure 4, we find that the θ crt increases with the number of magnet of type A, and then the θ crt reaches a maximum value of 35°. When we used different type of magnet, the maximum value of θ crt is nearly the same, θ crt is about 35°.

Exp. (2) Aspect ratio of coin.

Figure 5 shows that θ crt decreases from 30°–35° to 20° with increasing k, no matter which coin was used. The larger L and thinner D will significantly reduce θ crt.

Exp. (3) Distance between coin and magnet

From Figure 6, we find that the θ crt increases with distance at beginning, and then be a constant. The constant 0.05 at 90° which denoted by $\theta_{crt}$ means that the coin does not fall down. Even after being fully pulled down, the coin will reverse to original position at θ=0°. Finally the θ crt decreases.

Theoretical simulation and Discussion

Exp. (1) Strength of magnetic

Figure 7 shows the force acting on magnet. The solid lines indicate the magnetic flux.

Figure 7 shows the force acting on magnet. The force acting on magnet is divided into four parts denoted by arrows. The coin has been magnetized by the magnetic field of magnet. Assume top of magnet is N, and bottom of coin is S. F1 is the repelling force acting on north magnetic pole. F2 and F3 are the attracting force acting on south magnetic pole. The direction of torque of F1 and F2 are opposite, so we separate the force acting on south magnetic pole into F1 and F2.

When we gradually increase the magnetic field strength progressively (Increase the number of magnet.), as shown in left hand side of Figure 4, the magnetic force (F1, F2, and F3) become larger. But gravitational force is constant, then θ crt increases.

When the magnetic field is strong enough (In Figure 1, as H is above 1000 Oe, the coin is almost fully magnetized), the gravitational force can be ignored in comparison with magnetic force, the θ crt gradually saturates at about 35°. We do the Theoretical simulation for this result. When the coin is tilted by angle θ, we consider that the magnetization domain is determined by the diagonal line. See Figure 8(a). Assume that the distribution of magnetic moments is uniform. We calculate the torque of magnetic force theoretically. The calculation process is as follows.

$$\tau_x = x_1 \cdot M_1 \cdot H(y)$$
$$\tau_y = y_1 \cdot M_1 \cdot H(y)$$
$$\tau_z = z_1 \cdot M_1 \cdot H(y)$$

where

$$x_1 = \frac{M_1 \cdot A_1}{2 \cdot H(y)}$$
$$y_1 = \frac{M_1 \cdot A_1}{2 \cdot H(y)}$$
$$z_1 = \frac{M_1 \cdot A_1}{2 \cdot H(y)}$$

Finally the θ crt decreases.

Figure 7 shows the force acting on magnet. The solid lines indicate the magnetic flux.

Figure 8(b) shows the total torque as a function of tilting angle θ, and the value of θ crt when τ=0 is about 35°. It means that the gravitational force can be ignored in comparison with magnetic force, the θ c cartesian.
from 6 cm to 3 cm, the $\theta_{\text{crit}}$ increased. This behavior is similar with the result of experiment (1). (In experiment\(1\), $\theta_{\text{crit}}$ increases with the number of weak magnet.) When $H$ is below about 435 Oe, the magnetic force is weaker so the gravitational force can not be ignored. The increasing distance lead to the decreasing magnetic force $t$, then the critical angle decreases.

When the distance decreased from 3 cm to 0.5 cm, the $\theta_{\text{crit}}$ is constant. If $H$ is not strong enough ($H$ is between 435 Oe and 2080 Oe), magnetic moment of coin still arrange along the easy axis of coin, just magnetized by the partial component of $H$ along this direction.

And we consider two torques acting on magnet. One is induced by magnetic field acting on magnetic moment ($\theta_{1}$). The other is caused by gradient of magnetic field acting on magnetic moment ($\theta_{2}$). The calculations are as follows.

\[
\tau_{1} = M \times H = MH\sin\theta = \int (mdy)H\sin\theta
\]

\[
= \int_{y=0}^{y=\frac{L}{2}} m\frac{dy}{\cos\theta}H(y)\sin\theta - m\tan\theta \int_{y=\frac{L}{2}}^{y=L} H(y)dy
\]

\[\text{(note: } y = x\cos\theta, dM = mdx)\]

\[
\tau = F \sin\theta, F = M_{0}H = M\frac{dy}{\cos\theta}\cos\theta
\]

\[
\tau_{2} = x\sin\theta \cdot \frac{dH(y)}{dy}\cos\theta = \int_{y=0}^{y=\frac{L}{2}} x\sin\theta(mdx) \frac{dH(y)}{dy}\cos\theta
\]

\[\text{(note: } y = x\cos\theta, dM = mdx)\]

\[
\tau_{\text{net}} = \tau_{1} + \tau_{2} = M\sin\theta \cdot H(L\cos\theta)
\]

\[\therefore H(L\cos\theta) > 0\]

\[\therefore \tau_{\text{net}} > 0\]

Thus the coin never falls down.

Summary

In our experimental results, we find that $\theta_{\text{crit}}$ is strongly correlated with strength of magnetic field, aspect ratio of coin, distance between coin and magnet.

In experiment (1), we know that the $\theta_{\text{crit}}$ has a maximum about 35° due to the torque of magnetic force, and the magnetic force depends on strength of magnetic field. In experiment (2), $\theta_{\text{crit}}$ decreases with increasing $k$ because of unequal torque caused by different magnetic domain. In experiment (3), $\theta_{\text{crit}}$ increases with magnetic field when the field is below about 435 Oe. The coin never falls down when the field strength and gradient is proper. The increasing distance leads to the decreasing magnetic force $t$, so $\theta_{\text{crit}}$ increases with magnetic field.

By calculating the torques induced by magnetic field and field gradient, we can successfully explain why the coin can never fall down.

Summarizing those results, the key reason which affects $\theta_{\text{crit}}$ are strength of magnetic field, gradient of magnetic field, and distribution of magnetic domain.

Reference