2012 Problem 9 : Magnet and Coin

A magnetic coin's motion under a magnetic field

Abstract

We studied the motion of magnetic coins under a magnetic field, and investigated the key reasons that affected the critical angle θ_{crt}. When below or above θ_{crt}, the coin would reversed or fell down respectively. The strength of magnetic field, gradient of magnetic field, and distribution of magnetic domain are shown to be crucial to θ_{crt} value. We also find that the θ_{crt} has a maximum about 35°. Some theoretical calculations are also included for the explanation of experimental results.

Introduction

What is a magnetic coin? A magnetic coin is a metalic disc which contains magnetic substance, for example iron, nickel, and cobalt. When we applied a magnetic field, the coin would be magnetized. Well aligned magnetic moment would be attracted or repelled by magnetic field.
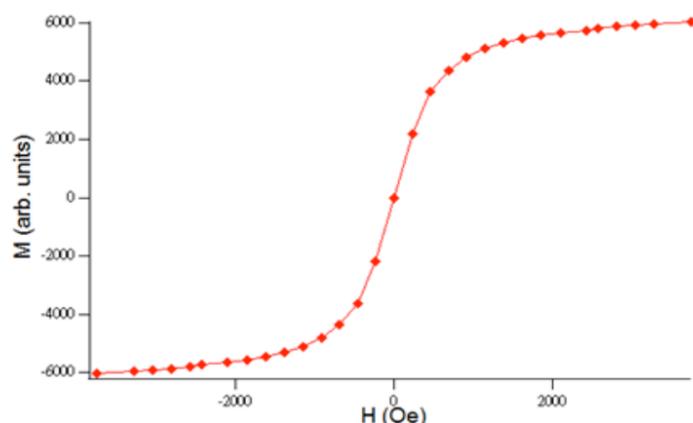
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Figure 1. The magnetic hysteresis loop of the coin used in our experiment. When H≥1000 Oe, the coin is almost fully magnetized.

Figure 1 shows the magnetic coin used in our experiment. It can be fully magnetized when the magnetic field is large enough (1000~2000 Oe). There is no remanent magnetization when H=0 Oe.

We study the motion of coins in the external magnetic field and discuss why the coin falls down or reverts. θ is the angle between the coin and the magnet surface normal, as shown in Figure 2(a). When the angle θ is larger than θ_{crt} (critical angle), the coin will fall down, otherwise it will revert. (Figure 2(b) and 2(c).)
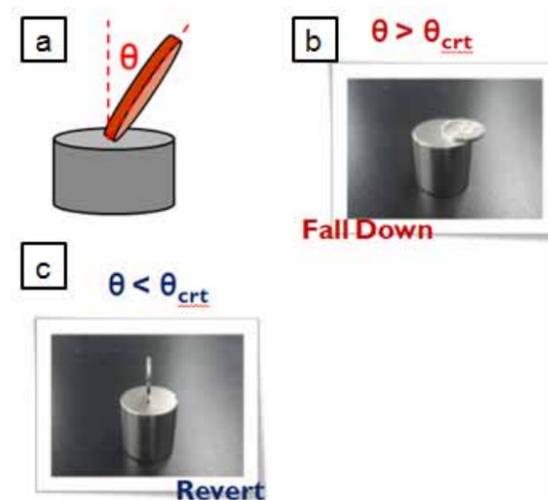
Figure 2. (a) The illustration of the angle θ. (b) The coin falls down when $\theta > \theta_{crt}$. (c) The coin reverts when $\theta < \theta_{crt}$

Experiment

Figure 3(a) shows the experimental setup. We used a straw to push the coin quasistatically. In order to study the variables that affect the critical angle, we change three parameters, which are (1) strength of magnetic field, (2) Aspect ratio of coin, and (3) distance between coin and magnet. We used a Gauss meter to measure the magnetic field of the magnet.

In Experiment (1), as shown in Figure 4, we used two kinds of magnets : (i) Strong magnet, H=3000 Oe or 5000 Oe and (ii) Weak magnet. We turned the H by number of magnet. The θ_{crt} increase from 13° to 35°.

In Experiment (2), as shown in Figure 5, we used two kinds of coin. One is made of pure steel. The other is made of ninety-five percent steel and five percent nickel. Each coin has different shape. The different shape means that coins have different aspect ratio k, which is the ratio of the diameter to the thickness.

$$k = \frac{L}{D}$$

L is the diameter of coin. D is the thickness of coin.

In Experiment (3), we changed the distance between coin and magnet, in order to change the magnetic field distribution, and gradient of magnetic field. We observed the corresponding changes of θ_{crt}.
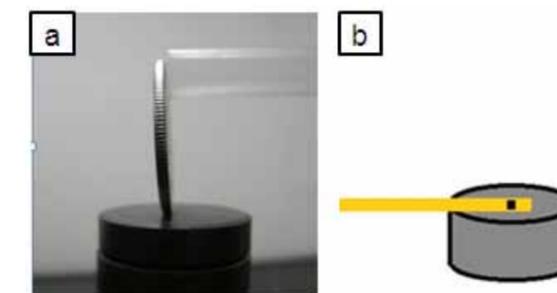
Figure 3. (a) The picture of the experimental setup. The coin is HK ten dollar. (b) The yellow crossbar is the sensor of Gauss meter which is used for the measurement of magnetic field.

Experimental Result
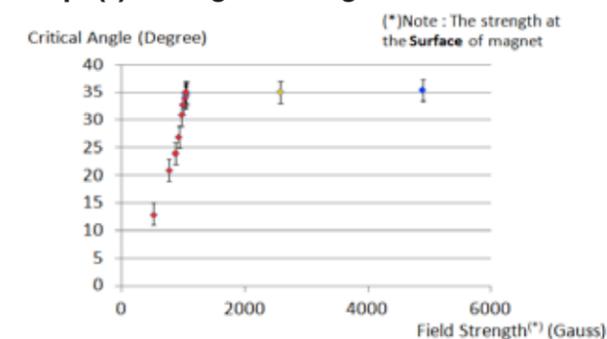
Exp. (1) Strength of magnetic field

Figure 4. θ_{crt} vs magnetic field strength by using different magnet. Each color of the dot represents each type of magnet.

In figure 4, the red dots represent the magnet of type A, yellow dot is type B, and blue dot is type C. The magnet of type A is the weakest magnet, and we used different number of magnet from one to eleven to increase the magnetic field. From figure 4, we find that the θ_{crt} increases with the number of magnet of type A, and then the θ_{crt} reaches a maximum value of 35°. When we used different type of magnet, the maximum value of θ_{crt} is nearly the same, θ_{crt} is about 35°.
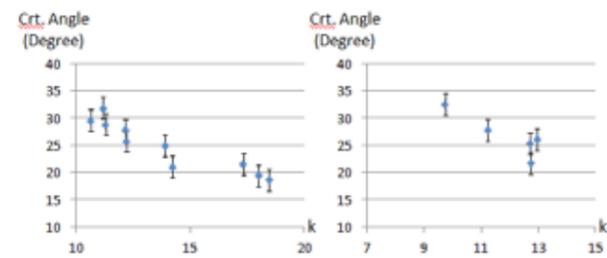
Exp. (2) Aspect ratio of coin.

Figure 5. θ_{crt} vs the aspect ratio for different kinds of coin. The picture in left is the coin made of 95% steel and 5 % nickel. The right side picture is the coin made of pure steel.

Figure 5 shows that θ_{crt} decreases from 30°–35° to 20° with increasing k, no matter which coin was used. The larger L and thinner D will significantly reduce θ_{crt}.
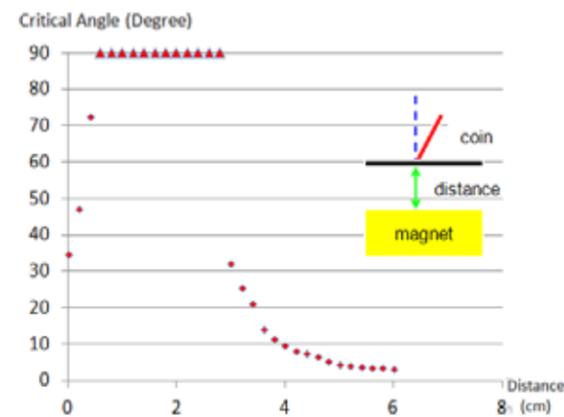
Exp. (3) Distance between coin and magnet

Figure 6. The θ_{crt} vs distance diagram.

From figure 6, we find that the θ_{crt} increases with distance at beginning, and then be a constant. The

constant θ_{crt} at 90° which denoted by ▲ means that the coin does not fall down. Even after being fully pulled down, the coin will reverse to original position at θ=0°. Finally the θ_{crt} decreases.

Theoretical simulation and Discussion
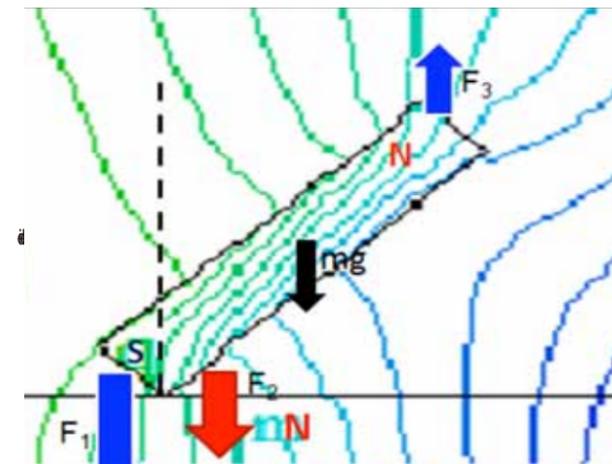
Exp. (1) Strength of magnetic

Figure 7. The illustration of the force acting on magnet. The solid lines indicate the magnetic flux.

Figure 7 shows the force acting on magnet. The force acting on magnet is divided into four parts denoted by arrows. The coin has been magnetized by the magnetic field of magnet. Assume top of magnet is N, and bottom of coin is S. F_3 is the repelling force acting on north magnetic pole. F_1 and F_2 are the attracting force acting on south magnetic pole. The direction of torque of F_1 and F_2 are opposite, so we separate the force acting on south magnetic pole into F_1 and F_2.

When we gradually increase the magnetic field strength progressively (Increase the number of magnet.), as shown in left hand side of Figure 4, the magnetic force (F_1, F_2 and F_3) become larger. But gravitational force is constant, then θ_{crt} increases.

When the magnetic field is strong enough(In Figure 1, as H is above 1000 Oe, the coin is almost fully magnetized), the gravitational force can be ignored in comparison with magnetic force, the θ_{crt}
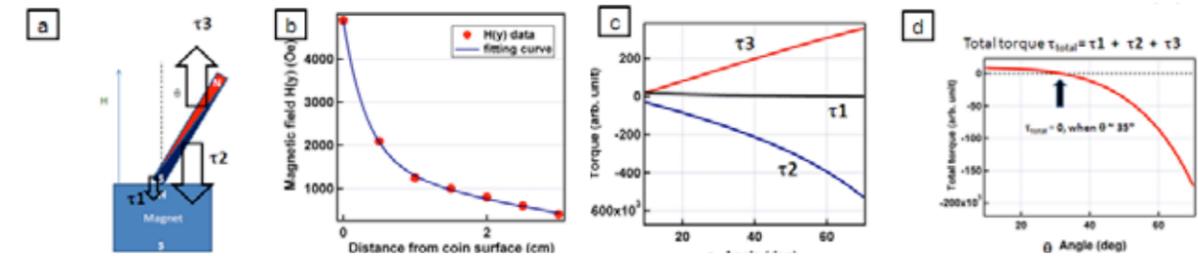

Figure 8. (a) The illustration of torques induced by different forces. (b) Magnetic field vs distance diagram. H(y) is a function of distance obtained by fitting the measured data. (c) The simulated diagram of torques τ1, τ2, and τ3 induce by F1, F2, and F3, respectively. (d) The simulated diagram of total torque which is a function of tilting angle θ.

gradually saturates at about 35° We do the Theoretical simulation for this result. When the coin is tilted by angle θ, we consider that the magnetization domain is determined by the diagonal line. See Figure 8(a). Assume that the distribution of magnetic moments is uniform. We calculate the torque of magnetic force theoretically. The calculation process is as follows.

$$\tau_1 = x_1 \cdot M_1 \cdot H(y_1)$$
$$= m(D\cos\theta/3)\cdot[D\cos\theta(D\sin\theta+D\cos\theta\cot(\alpha+\theta))/2]\cdot H(y1)$$
$$y_1 = (D\cos\theta\cot(\alpha+\theta)+2D\sin\theta)/3$$

$$\tau_2 = x_2 \cdot M_2 \cdot H(y_2)$$
$$= m(L\sin\theta/3)\cdot[L\sin\theta(D\sin\theta+D\cos\theta\cot(\alpha+\theta))/2]\cdot H(y2)$$
$$y_2 = (D\cos\theta\cot(\alpha+\theta)+D\sin\theta+L\cos\theta)/3$$

$$\tau_3 = x_3 \cdot M_3 \cdot H(y_3)$$
$$x_3 = [A'(2L\sin\theta-D\cos\theta)+A''L\sin\theta]/3(A'+A'')$$
$$M_3 = m(A'+A'')$$
$$y_3 = [A'(2L\cos\theta+2D\sin\theta+D\cos\theta\cot\theta)+A''(L\cos\theta+2D\sin\theta+D\cos\theta\cot\theta+D\cos\theta\cot(\theta+\alpha))]/3(A'+A'')$$
$$A' = L\sin\theta[D\cos\theta\cot\theta-D\cos\theta\cot(\theta+\alpha)]/2$$
$$A'' = D\sqrt{(L\sin\theta-D\cos\theta)^2+(L\cos\theta-D\cos\theta\cot\theta)^2}/2$$
$$H(y) = -822+3239e^{-y/0.322}+2485e^{-y/4.4}$$

tanα=D/L. M is magnetic moment of coin. In Figure 8(b), H(y) is magnetic field already measured as a function of the distance to the magnet surface. We measured that D (The thickness of coin) is 2.2 mm and L (The diameter of coin) is 2.5 mm.

Figure 8(d) shows the total torque as a function of tilting angle θ, and the value of θ_{crt} when τ=0 is about 35°. It is consistent with our experiment results(See Figure 4).
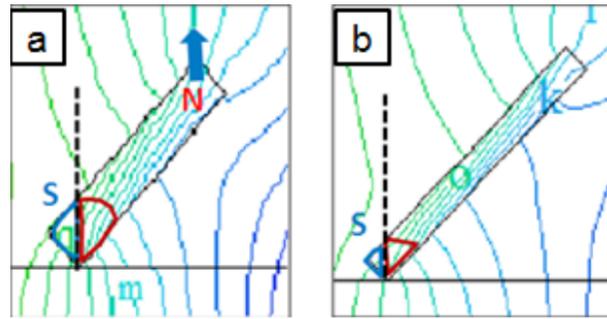
Exp. (2) Aspect ratio of coin

Figure 9. The illustration of magnet in different k. (a) The coin on the left hand side has smaller k. (b) The coin on the right hand side has larger k.

Figure 5 indicate that θ_{crt} decreases with increasing k. It means that the coin on the left hand side in Figure 9 has larger θ_{crt} . Why? As we see in figure 9(a), the magnetic domain in the left side of the dotted line is larger than figure 9(b). When we increase k, the magnetic domain in the left side of the dotted line becomes smaller, so the torque of force F1 is decrease with increasing k. It is the reason why θ_{crt} decreases with increasing k.
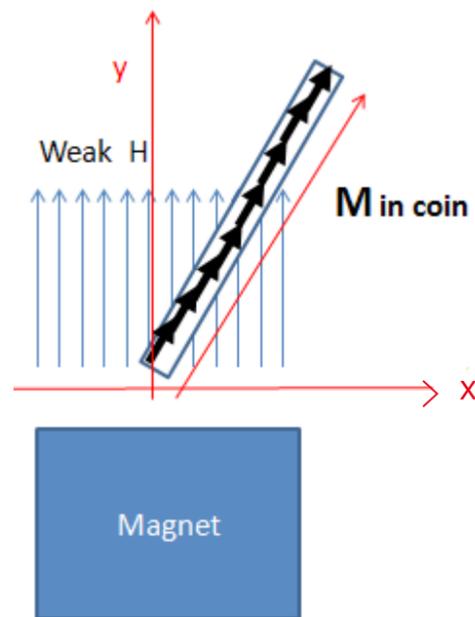
Exp. (3) Distance between coin and magnet

Figure 10. The illustration of the coin which never fall down.

As we see in figure 6, when the distance decreased

from 6 cm to 3 cm, the θ_{crt} increased. This behavior is similar with the result of experiment (1). (In experiment1, θ_{crt} increases with the number of weak magnet.) When H is below about 435 Oe, the magnetic force is weaker so the gravitational force can not be ignored. The increasing distance lead to the decreasing magnetic force t, then the critical angle decreases.

When the distance decreased from 3 cm to 0.5 cm, the θ_{crt} is constant. If H is not strong enough (H is between 435 Oe and 2080 Oe), magnetic moment of coin still arrange along the easy axis of coin, just magnetized by the partial component of H along this direction. And we consider two torques acting on magnet. One is induced by magnetic field acting on magnetic moment (1). The other is caused by gradient of magnetic field acting on magnetic moment (2). The calculations are as follows.

$$\tau_1 = M \times H = MH\sin\theta = \int_{x=0}^{x=L}(m\,dx)H\sin\theta$$

$$= \int_{y=0}^{y=L\cos\vartheta} m\frac{dy}{\cos\theta}H(y)\sin\theta = m\tan\theta\int_{0}^{L\cos\theta}H(y)dy$$

$(note: y = x\cos\theta, dM = mdx)$

$\tau = F\ x\ \sin\theta,\ F = M\cdot\theta H = M\cdot dH(y)/dy\cdot\cos\theta$

$$\tau_2 = x\sin\theta\cdot M\frac{dH(y)}{dy}\cos\theta = \int_{x=0}^{x=L}x\sin\theta(m\,dx)\frac{dH(y)}{dy}\cos\theta$$

$$= \int_{y=0}^{y=L\cos\vartheta}\frac{y}{\cos\theta}\sin\theta\cdot m\frac{dy}{\cos\theta}\frac{dH(y)}{dy}\cos\theta$$

$$= m\tan\theta\int_{0}^{L\cos\theta}y\frac{dH(y)}{dy}dy$$

$$= m\tan\theta\cdot y\cdot H(y)\Big|_{0}^{L\cos\theta} - m\tan\theta\int_{0}^{L\cos\theta}H(y)dy$$

$(note: y = x\cos\theta, dM = mdx)$

$\tau_{total} = \tau_1 + \tau_2 = M\sin\theta\cdot H(L\cos\theta)$
$\because H(L\cos\theta) > 0$
$\therefore \tau_{total} > 0$

Thus the coin never falls down.

Summary

In our experimental results, we find that θ_{crt} is strongly correlated with strength of magnetic field, aspect ratio of coin, distance between coin and magnet. In experiment(1), we know that the θ_{crt} has a maximum about 35° due to the torque of magnetic force, and the magnetic force depends on strength of magnetic field . In experiment(2), θ_{crt} decreases with increasing k because of unequal torque caused by different magnetic domain. In Exp. (3), θ_{crt} increases with magnetic field when the field is below about 435 Oe. The coin never falls down when the field serength and gradient is proper. The increasing distance leads to the decreasing magnetic force t, so θ_{crt} increases with magnetic field.

By calculating the torgues induced by magnestic field and field gradient, we can successfully explain why the coin can never fall down.

Summarizing those results, the key reason which affects θ_{crt} are strength of magnetic field, gradient of magnetic field, and distribution of magnetic domain.

Reference

[1] http://www.youtube.com/watch?v=qnQTXHo6OHo

[2] http://www.youtube.com/watch?v=Llcn7VjOrjc

[3] J. D. Jackson. Classical Electrodynamics (3rd ed., John Wiley & Sons, 1998)

[4] H. Goldstein. Classical Mechanics (3rd ed. Addison-Wesley, 2002)