2010 Problem 11 : Sand

What Determines Its Softness

Abstract

To investigate the parameters that affect the softness of the sand, "softness" is defined as the reciprocal of hardness. A method is introduced to measure the softness by dropping a ball into the sand and compare the kinetic energy losses through the same distance below the sand level. The velocities are measured by analyzing the high-speed video clips frame by frame. Theoretical explanation is provided and justified by experiment.

Parameters are found and carefully examined. The major one is the water concentration. Other parameters such as the size of sand particles and sand-water-air ratio are also important.

Keywords

sand softness, water concentration, sand size, liquid bridge, saturation, adhesion force

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2010 IYPT Problem 11: Dry sand is rather "soft" to walk on when compared to damp sand. However sand containing a significant amount of water becomes soft again. Investigate the parameters that affect the softness of the sand.

I.Definition of softness

In order to investigate the parameters that affect the softness of damped sand, it is necessary to define "softness" to measurable variables. "Softness" is an ambiguous term that has been rarely defined; however, there are several common ways to determine the "hardness" of an object. Hardness refers to various properties of matter in the solid phase that gives it high resistance to various kind of shape change when force is applied.

There are traditionally two methods to measure hardness[1]. First, scratch one object with another and compare their surface; the one with scratches is less hard (or softer) than the one without. This method relies on the friction between the two objects. The second most commonly applied method measures indentation or deformation due to a constant load.

A third method is sometimes feasible for certain objects that deform drastically when force is applied. Release two objects at the same height into free fall motion and measure the height of the bounce by each object. The hardness measured using this method is related to the elasticity of the objects.

The hardness of wet sand is relevant to friction, adhesion, density and other factors. Therefore, traditional methods, both static and dynamic ones are hard to apply. The applicable method is to combine the two.

Use an iron ball and make it free fall from a height H above the sand level. It will sink to a certain depth into the sand. Assume that the resistant force is proportional to hardness; therefore, for sand of different water content, the forces only differ by a factor k

$$F(x) = kF_0(x) \qquad (1)$$

where $\boldsymbol{x}$ is the depth the ball sinks below the sand and $\boldsymbol{F_0}$ is the resistance force, a function against the depth, in dry sand sample. Therefore, $\boldsymbol{k}$ represents the hardness of the sand in a given sand sample. Once the hardness of the sand is defined and becomes measurable, softness can be easily defined as the reciprocal of the hardness $\boldsymbol{k}$.

$$\boldsymbol{S} = 1/\boldsymbol{k} \qquad (2)$$

Take the dry sand as a standard to compare with wet sand, and set the softness of dry sand as 1. For dry sand, the work done by the resistant force from the top of the sand level to at the depth $\boldsymbol{h}$ below is:

$$W_0 = \int_0^h F_0\,dx = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \qquad (3)$$

For wet sand:

$$W = \int_0^h F\,dx = \frac{1}{2}mv'^2 - \frac{1}{2}mv_0^2 \qquad (4)$$

Here v0 is the initial velocity of the ball just hitting the sand, v and v' are velocities at depth h (h has to be fixed for different sand samples).

From (1)-(4), the relative softness can be defined as

$$S = \frac{1}{k} = \frac{W_0}{W} = \frac{v_0^2 - v^2}{v_0^2 - v'^2} \qquad (5)$$

As long as one can measure the velocity of ball at a certain depth, one can easily determine the softness.

II.Experiment

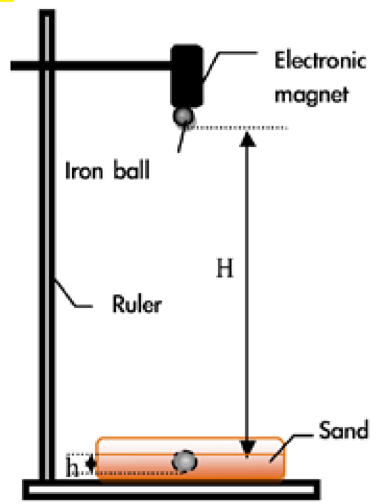

FIG.1 Experimental setup

The device is shown in fig.1. The setup includes an iron support, a ruler, a slide caliper, an electronic magnet, several circular containers of the same size, iron balls, and sand samples. A container filled with sand is placed on the bottom of the support. The electronic magnet is fixed on a movable settle that will ensure smooth adjusting. An iron ball can hang at the end of the magnet when it is turned on. Additional measuring devices including an electronic balance and a glass measuring cup are necessary for determining the initial conditions of the sand. The speed of the ball is measured by using a high speed camera (Casio FC150, 420fps) and analyzing the video clips frame by frame.

In the experiment, the sand samples have the density of 1.56g/cm3. The iron balls are identical with mass 27.5g and diameter 19mm.

FIG. 2a Each box of sample is used for ONCE

FIG. 2b Identical dry sand samples to be mixed with different amount of water

To examine the relationship between water-sand ratio and sand softness, prepare identical dry sand samples to be mixed with different amount of water (see fig.2). In the experiment conducted in this research, each box of sample is 1120g + 10g and is only used for ONCE. Water is uniformly poured on the sand surface using filter papers. The position of the iron ball against time is determined by using QuickTime® and Photoshop® (see fig. 3) In this way, the velocities can be easily determined.

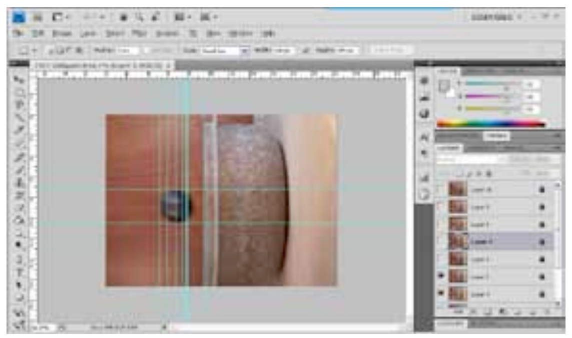

FIG. 3 The position and time relation is determined by using QuickTime and Photoshop

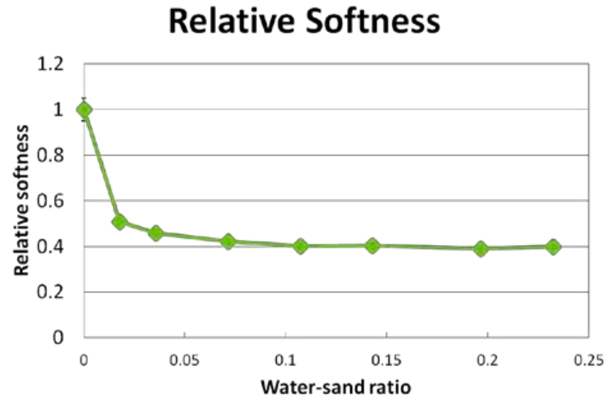

FIG. 4 Relative softness is measured as a function of water content

The velocities are recorded and calculated into the relative softness according to equation (5); the relative softness is measured as a function of water content. Define water-sand ratio as the ratio of the mass of water poured in to the mass of the entire sand sample. The results are shown in fig. 4. A very obvious soft-to-hard transition is observed before the saturation point, the point at which sand-water ratio is approximately 0.2. The hard-to-soft transition after saturation is more difficult to observe in this figure, because the harder the sand sample, the smaller decrease Δ v' in the iron ball velocity at depth ***h***. When calculating the softness ***S*** using equation (5), this change is further diminished by taking the square of v'. However, when observing this phenomenon qualitatively, instead of using softness ***S***, one can use the final depth of the iron ball when its motion comes to a full stop. The change in the final depth is more obvious than the change in the velocity at depth ***h***; thus the saturation point and the hard-to-soft transition are more obvious in fig. 5.

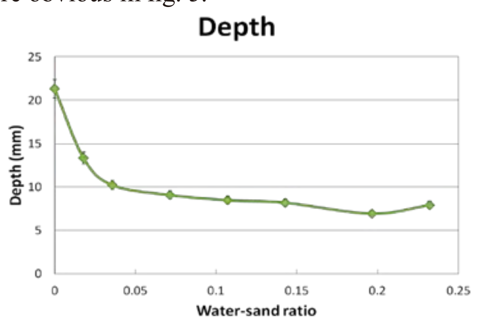

FIG. 5 Depth is a good measure of softness. The depth the ball sinks down under the sand level after falling from a height of 40cm is measured as a function of water content.

In the experiment process described above, however carefully one pours the water, the water distribution in the sand samples cannot be even. It is thus reasonable to consider stirring the samples after adding water to ensure that the sample is uniform in wetness. Fig.6 shows the results from a stirred sample and a non-stirred sample respectively.

The two methods, stirring and non-stirring, share some similarities while encounter more differences. When adding water without stirring the sand, the hardness of sand increases as water-to-sand ratio rises until the sand reaches its saturation; when the sand is stirred, no significant change can be observed in softness as the water-sand ratio rises. The stirred wet sample remains soft because it contains air that breaks liquid bridges to different extent depending on the force and method of stirring.

Only when the water is saturated does the sand in stirred samples suddenly turn hard. At the saturation point, the depth of the ball, ***h***, is a constant, and accordingly the sand shares the same hardness when saturated under whichever condition. Water molecules are no longer separated by sand particles in saturated sand samples; therefore, even after stirring, the water layer, which is now continuous, will drive the air away. After the water level surpasses the sand level, the sand becomes slightly softer under both conditions.

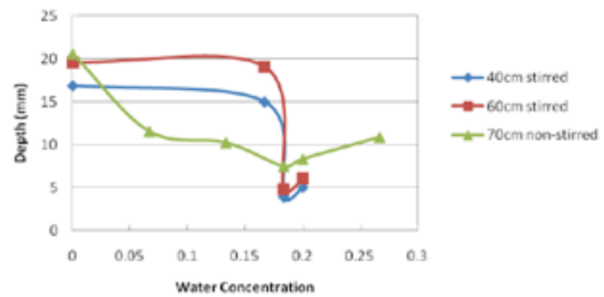

FIG. 6 Compare the softness under two different conditions: one is to add water without stirring the sand, the other is to add water and stir to make it uniform in wetness. The ball falls from the height of 40cm/60cm/70cm respectively.

From fig. 6, the hard-to-soft transition is similar under both conditions. The soft-to-hard transition is more apparent than the hard-to-soft transition, though process in the non-stirring sample is much more gradual.

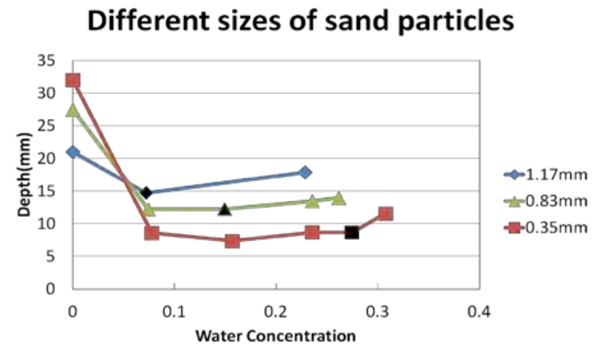

FIG. 7 The depth vs. water content relation is measured for sand of different sizes. Smaller sand tends to have more significant change in softness.

Finally, to investigate the influence of the size of sand particles, separate the sand into samples of three different sizes by meshes with hole diameters of 1.17mm, 0.83mm, and 0.35mm. Fig. 7 shows the depth of the iron ball when it comes to a full stop in the three different-sized sand samples; the balls fall from 40cm above the sand surface. As one can see, the samples that are constituted of smaller sand particles have more significant change in softness, which will be explained by the following theory.

III.Theory

FIG. 8 Sand has three states: dry sand, sand with less water, and sand with excess water

Sand has three states: dry sand, sand with lesser water, and sand with excess water. At each state, the major factors that affect the softness differ because the interaction between sand and water is different (see fig.

8.). When the sand is dry, normal force and friction are dominant. When the water-sand ratio is below saturation, the adhesion force produced by liquid bridge[2,3] is dominant. When the sand is totally saturated, the sand is completely merged in the water, and the liquid bridge disappears. The transitions between states are highly related to the change in sand softness.

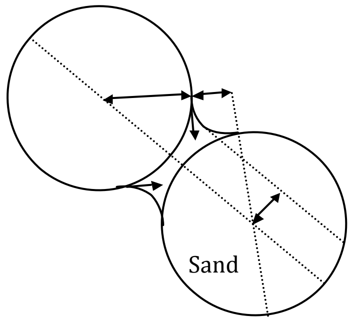

FIG. 9 Liquid bridge is formed between sand particles

The adhesion force produced by liquid bridge is the main cause of the gradual decrease in softness in unsaturated wet sand. In the former experiment where stirring is not applied, a relatively apparent and gradual change in softness can be observed. When water is added, liquid bridges form between the sand particles. A sand-water model (see fig. 9) can be set up to investigate the relation between the amount of water added and the adhesion force.

Liquid bridge mainly provides the adhesion force f_L [2]

$$f_L = \pi r_2^2 \gamma \left(\frac{1}{r_1} - \frac{1}{r_2}\right) + 2\pi r_2 \gamma \qquad (6)$$

where γ is the surface tension, r_1 and r_2 are radii of curvatures. The first term in the right hand side arises from the reduced Laplace pressure. The second term is due to the surface tension.

The equation (6) can be simplified as

$$f_L = \frac{\pi r_2^2 \gamma}{r_1} - \pi r_2 \gamma + 2\pi r_2 \gamma = \frac{\pi r_2^2 \gamma}{r_1} + \pi r_2 \gamma \qquad (7)$$

Because the distance between the two sand particles are very small, the liquid bridge will almost be cylindrical. The curvature radius of liquid bridge r_1 is significantly larger than r_2. So we can reduce the equation into the following.

$$f_L = \pi r_2 \gamma \qquad (8)$$

Within a certain range, the volume of the mixture does not change significantly. Assume that the distance between sand particles remains constant within a range, then

$$V_{water} \propto S \propto \pi r_2^2 \propto r_2^2 \qquad (9)$$

S represents the contacting area between sand and water. From (8) and (9), the adhesion force f_L is supposed to be proportional to the square root of the water volume.

$$f_L \propto \sqrt{V_{water}} \qquad (10)$$

In reality, the slight change in volume of the mixture exists, so

$$f_L \propto V_{water}^n \left(1/3 < n < 1/2\right) \qquad (11)$$

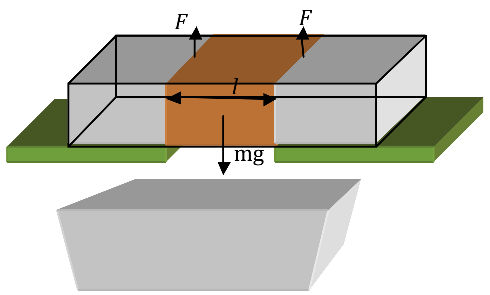

FIG. 10 The experiment setup to measure the adhesion force caused by liquid bridge

This theory can be justified by a simple experiment. An open box (9cm×16cm×2.5cm), several boards and a container are constructed as in fig. 10. The box is filled with uniform wet sand and put upside-down on two boards. The two boards are moved sideways, so that the sand in the middle is suspended by adhesion between water and sand. A ruler is placed parallel to the box side to measure the maximum distance that can support the sand inside.

All the force is provided by adhesion force on the cross-section surface. The sand block drops when the adhesion force on cross-section is not big enough to hold all the sand inside. By measuring the mass of the sand dropped, we can learn the magnitude of adhesion force. So the adhesion force F caused by liquid bridge satisfies

$$2F = mg \qquad (12)$$

With the weight of the dropping sand, we can calculate the adhesion force per unit area. In fig.11, we can get the relation between F/S and water volume. For loose sand

$$F/S \propto V^{0.3783} \qquad (13)$$

for compact sand

$$F/S \propto V^{0.3949} \qquad (14)$$

From the equation (13) and (14), the F/S is to 0.37-0.39th power, in agreement with equation (11).

The effect of the liquid bridge is to strengthen the sand, that is to say, to reduce the softness of the sand. This explains the tendency that sand turns hard when more water is added. The insignificant change in stirred sample also shows when liquid bridges are broken; the strengthening effect is also reduced.

According to this liquid bridge model, sand samples with smaller sand particles tend to have more significant change in softness. The adhesion force between water and sand is relatively strong for smaller one, because smaller one has more surface area to volume ratio.

After the saturation point, water level surpasses the sand level, making top of the mixture fluid. The top layer of sand is floating in the water, making them loose and disconnected with each other. The shearing strength between sand is similar to that of the fluid, much smaller than the adhesion force created by liquid bridge. So exceeding amounting of water can soften the sand again.

IV.Conclusion

The softness of the sand is determined mainly by water concentration. The saturation point, or the point at which adding more water does not create more or strengthen current liquid bridge, is a critical point in this experiment. Before saturation, a gradual soft-to-hard transition occurs when the sand is mixed with water and without stirring; the sand remains soft when it is stirred and its softness largely depends on the air content in the sample. The hardening effect is mainly caused by the liquid bridges, which are created when more water is added. At the saturation point, however, both stirred and non-stirred samples experience an increase in hardness. After the sand sample is saturated, when excessive amount of water is added into the sample, the surface layer of water is fully "connected," the top layer of sand starts to float, and the liquid bridges disappear. At this stage, the dominant force is wet friction, which is weaker than the adhesion force produced by the liquid bridges. Other parameters, such as the size of sand particles and the amount of air in a damped sand sample can also affect the softness of the sand.

Reference

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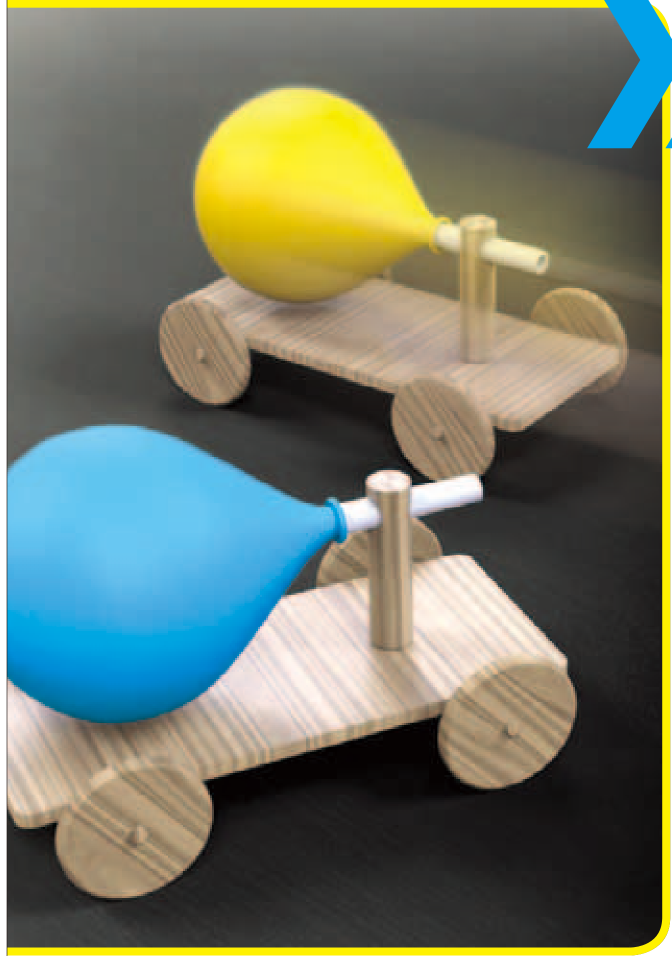

2010 Problem 5 : Car
Car powered by a balloon

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Abstract

This article mainly solved the distance traveled by the car which powered by a spherical balloon, an equation of relevant parameters to determine the distance was derived. In experiment part, a toy car is designed. Three variables were controlled, the radius of the balloon, the radius of the vent and the angle of the vent inclination. The distance increased exponentially with the increment of radius. There is a maximum value of distance when the radius of vent increasing. The distance decreased with the increment of the angle. The result of the experiment fitted the theory well.

Keywords

car, balloon, distance.

Introduction

A car powered by a balloon is a very attractive problem among students all around. This kind of problem is also list in IYPT before. The distance it travels depends on many parameters such as balloon, gears, wheels and so on. Many people studied the problem of the inflation, deflation and reinflation of a single balloon[2].The pressure in the balloon when inflating and deflating[3]. The damage and the healing of the balloon when inflating and deflating[4]. They also studied the difference between the spherical and cylindrical balloon[5]. The elastic instability of balloon is also discussed[6]. But few of them have analyzed the problem of the car. In order to solve the problem the car is divided into two parts, one is the power part, the other is the resistance part. We can find the balloon here is very important, because the balloon is the only thing that provides power.

Theory for car

The stress exerts on the balloon is defined by(1a), the thickness of